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A New Minimal Detectable Bias in Fault Detection for Positioning

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Abstract

The Minimal Detectable Bias method of Fault Detection is frequently employed to determine if a position has integrity. However, to provide integrity the Type I error probability of the statistical tests is required to be preset. Normally, this probability is set to avoid the unnecessary rejection of measurements or to satisfy the continuity requirements. In this paper, the Type I error probability is set based on the integrity requirements by initially setting the Protection Levels equal to the Alert Limit. This new procedure of setting the Type I error probability is compared with the more conventional approach when there are different continuity requirements and when multiple biases are considered. From the results of this comparison, it is concluded that the new procedure increases the availability rates regardless of the continuity requirements and the number of biases considered.

Keywords: Integrity, Continuity, Availability, Multiple Biases

1. Introduction

The concept of integrity is based on the user specifying the maximum amount of positioning error tolerated which is commonly referred to as the Alert Limit. In addition to the Alert Limit, the user is also required to set the Probability of a Missed Detection, which is the maximum probability of the position being in error greater than the Alert Limit that will be forgiven by the user. If the reported position is in error greater than the Alert Limit more frequently than the Probability of a Missed Detection, then the user is no longer forgiving and considers that the integrity of the position has been lost.

Whilst the Minimal Detectable Bias (MDB) method of Fault Detection, based on either the chi-square test (Baarda, 1967; Parkinson and Axelrad, 1988; Sturza, 1988; Brown, 1992) or the outlier test (Baarda, 1968; Kelly, 1998), can be used to determine if a position has sufficient integrity for a given Alert Limit. The procedure is also dependent on the Type I error probability of the statistical tests being preset in order to determine the thresholds for the statistical tests and the Protection Levels. Therefore, the question arises as how to set the Type I error probability of the statistical tests?

In applications with continuity requirements, such as aviation, the Type I error probability is traditionally set to always satisfy the continuity requirements. Therefore, to determine if the positioning system satisfies both continuity and integrity requirements, that is available, it is only required to monitor the position's Protection Level (Ober, 2000b). Since the applications that have continuity requirements are adverse to continuity risks, the Type I error probabilities that are set are very small. Hence, the systems are inadvertently adverse to the rejection of measurements.

In some applications, such as geodesy, where the measurements are remeasured if they are rejected by the statistical tests, the Type I error probability is set to avoid such rejections. Since it is reasoned that due to the high cost of remeasurement, the Type I error probability should be set to avoid such unnecessary rejections. Therefore, the Type I error probability in geodesy is typically set to 1%, or 0.1%, such that only 1 in 100, or 1 in 1000, measurements are unnecessarily rejected (Baarda, 1968).

The adverse impact of setting a small Type I error probability, which avoids the rejection of measurements, is that it increases the Protection Level. Therefore, by increasing the Type I error probability, the Protection Level can be reduced and the position can gain integrity. However, this is at the expense of the continuity probability and an increased probability of rejecting measurements. Nevertheless, in applications that do not have continuity requirements, or remeasure, this appears to be a feasible strategy of increasing the percentage of time that a position with integrity can be obtained.

Another aspect of initially setting the Type I error probability is that it also often results in the position's Protection Level being less than the Alert Limit. If the Type I error probability was increased, then the position's Protection Level could be made equal to the Alert Limit. Meaning that a position with integrity is still obtained, but with a reduced continuity risk and a reduced probability of measurements being rejected.

By initially setting the same Type I error probability, and the same Probability of a Missed Detection, for each measurement there are numerous different Protection Levels that are obtained. To obtain the position's Protection Level, it is then conservatively assumed that the bias always corresponds with the most difficult to detect measurement that produces the largest Protection Level (Lee et al., 1996; Lee and Van Dyke, 2002). Whilst this is a conservative assumption, it is at the cost of availability. In an attempt to address this, Lee and Van Dyke (2002) assume that the bias can exist in any one of the measurements and average the Missed Detection Probabilities across all of the measurements. However, it was found that this only results in a minor improvement in availability. If the conservative assumption is maintained, then when a statistical test fails due to a measurement that does not correspond with the largest Protection Level it may not actually be a significant Missed Detection at all. Whilst this is the only option when the chi-square test is employed, with the outlier test it is possible make all of the measurements have the same Missed Detection Probability and the same Protection Level by changing the Type I error probabilities. Such setting of the Type I error probabilities will also result in a reduction in the probability of a measurement being rejected and a reduction in the continuity risk.

Besides the current procedure of applying the MDB method that requires the Type I error probability to be preset some, Fault Detection methods exist that initially satisfy the integrity requirements and then monitor the continuity risk. In general, these methods tend to be position domain techniques whereas the methods that require the Type I error probability to be preset are measurement domain techniques (Ober, 2000b). They include the multiple hypothesis method (Pervan et al., 1998; Blanch et al., 2007) and the Bayesian approach (Ober, 2000a).

In the multiple hypothesis method and the Bayesian approach, it is expected that the positioning solution using all of the measurements is equal to the positioning solution with one of the measurements removed. If a biased measurement is present, then the probability density function of the solution using all of the measurements is expected to be biased while the solution with the biased measurement removed is expected to correspond with the true position. Therefore, by comparing the difference between all the positioning solutions, and their distributions, with the Alert Limit it can be determined if the positioning solution is unreliable. However, in the multiple hypothesis method the distributions are weighted by their a priori probabilities while the Bayesian approach weights the distributions by their a posteriori probabilities. The problem with weighting the distributions with their a posteriori probabilities is that the continuity probability cannot be predicted (Ober, 2000b). Nevertheless, comparing the multiple hypothesis method with the Bayesian approach Ober (2000a) concludes that the multiple hypothesis method produces optimistic estimates to the Missed Detection Probability. The main problem with both methods though is that when they are extended to two or more dimensions numerical integration of the probability density functions is required. Therefore, they are generally not practical methods of providing integrity.

Hence, this paper persists with the MDB method rather than employing any of the existing position domain techniques. The procedural operation of the MDB method though is changed to set the Protection Level of each measurement equal to the Alert Limit by changing the Type I error probabilities. Therefore, placing integrity as the first priority and simply maximising the continuity probability with respect to integrity. In addition, the developed operational procedure of the MDB method is also extended to the case of two biases.

2. Fault Detection and Exclusion For a Single Bias

2.1 The Conventional MDB Procedure

In the conventional procedure of applying the MDB method, using the outlier test, the continuity probability is initially set. Then, using the continuity probability the Probability of a False Alert, P_{FA}, is obtained, and the Type I error probabilities of the outliers tests are also obtained as (Sĭdák, 1968; Kelly, 1998)

$$\alpha_i = 1 - \sqrt[n]{1 - P_{FA}} \tag{1}$$

where n is the number of measurements. Therefore, the presence of a bias can be detected with the outlier statistic (Baarda, 1968; Kelly, 1998)

$$w_i = \frac{\mathbf{h}_i^{T}\mathbf{P}\mathbf{Q}_{\mathbf{v}}\mathbf{P}\ell}{\sigma_0\sqrt{\mathbf{h}_i^{T}\mathbf{P}\mathbf{Q}_{\mathbf{v}}\mathbf{P}\mathbf{h}_i}} \sim N(0,1)_{1-\alpha_i/2} \tag{2}$$

where $\mathbf{P}$ is the weight matrix, σ_0 is the a priori scale factor, $\mathbf{Q}_{\mathbf{v}}$ is the cofactor matrix of the estimated residuals, $\mathbf{h}_i$ is a vector of zeros with a one in the i^{th} entry, and ℓ is the measurement vector. If one or more of the outlier tests fails, then it is deduced that there are one or more bias measurements. In the case of Fault Detection and Exclusion (FDE), the measurement that corresponds with the largest outlier statistic is rejected and the outlier test is reapplied. This is continued until all the outlier statistics pass, or there are an insufficient number of measurements remaining.

Even if all of the outlier statistics pass, there is still a possibility of a bias going undetected that causes the positioning solution to be in error greater than the Alert Limit. To ensure that the probability of such an event is less than the Probability of a Missed Detection, Protection Levels are formulated to indicate the region in which this is not the case. In the MDB method, it is initially assumed that the Probability of a Missed Detection, P_{MD}, is equal to the Type II error probability, β_i, of the outlier test (Kelly, 1998). With the set Type I and Type II errors the corresponding shift in the outlier statistic is normally approximated as (Baarda, 1968; Kelly, 1998)

$$\delta_0 = \mathrm{N}(0,1)_{1-\alpha_i/2} - \mathrm{N}(0,1)_{\beta_i} . \tag{3}$$

While this is reasonable when the probabilities are small (Oliveira and Tiberius, 2009), it becomes increasingly errorous with larger probabilities. Conversely, the correct δ_0 can be obtained by converting the normal distributions to chi-square distributions, as shown in Fig. 1, which yields

$$\chi^2_{1-\alpha_i,1} = \chi^2_{\beta_i,1,\delta_0{}^2} . \tag{4}$$

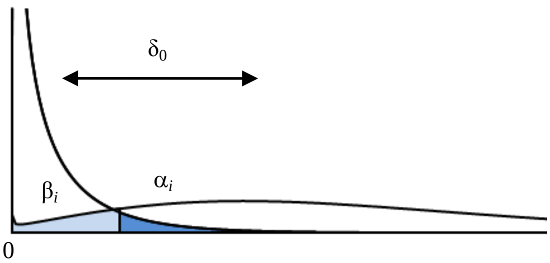

Figure 1: Chi-Squared Distributions of the Null and Alternate Hypotheses

In the presence of a bias though the expected shift in the i^{th} outlier test is given by (Baarda, 1968; Kelly, 1998)

$$\delta_i = \frac{\sqrt{\mathbf{h}_i{}^{\mathrm{T}}\mathbf{PQ}_{\mathbf{v}}\mathbf{Ph}_i}\,\nabla s_i}{\sigma_0} \tag{5}$$

where ∇s_i is the bias in the i^{th} measurement. Therefore, on substitution of δ_0 the MDB can be obtained as (Baarda, 1968; Teunissen, 1990; Kelly, 1998)

$$\nabla_0 s_i = \frac{\delta_0\sigma_0}{\sqrt{\mathbf{h}_i{}^{\mathrm{T}}\mathbf{PQ}_{\mathbf{v}}\mathbf{Ph}_i}} . \tag{6}$$

To determine the impact of the MDB on the final position, it is initially required to notice that the expected shift in the least squares solution caused by a bias is given by (Baarda, 1968; Kelly, 1998)

$$\nabla\mathbf{x}_i = (\mathbf{A}^{\mathrm{T}}\mathbf{PA})^{-1}\mathbf{A}^{\mathrm{T}}\mathbf{Ph}_i\nabla s_i \tag{7}$$

where $\nabla\mathbf{x}_i$ is a t by one vector and $\mathbf{A}$ is the design matrix. Therefore, on substitution of Eq. (6) the impact of the MDB on the least squares solution is given by

$$\nabla_0\mathbf{x}_i = \left(\mathbf{A}^{\mathrm{T}}\mathbf{PA}\right)^{-1}\mathbf{A}^{\mathrm{T}}\mathbf{Ph}_i \frac{\delta_0\sigma_0}{\sqrt{\mathbf{h}_i{}^{\mathrm{T}}\mathbf{PQ}_{\mathbf{v}}\mathbf{Ph}_i}} . \tag{8}$$

With an appropriately constructed **C** matrix, to select the coordinates of interest, the Protection Level for the i^{th} measurement can be obtained as (Chin et al., 1992; Brown and Chin, 1997; Angus, 2006; Wang and Kubo, 2010)

$$\mathrm{PL}_i = \sqrt{\nabla_0\mathbf{x}_i{}^{\mathrm{T}}\mathbf{C}^{\mathrm{T}}\mathbf{C}\nabla_0\mathbf{x}_i} , \tag{9}$$

which becomes

$$\mathrm{PL}_i = \sqrt{\frac{\mathbf{h}_i{}^{\mathrm{T}}\mathbf{PA}\left(\mathbf{A}^{\mathrm{T}}\mathbf{PA}\right)^{-1}\mathbf{C}^{\mathrm{T}}\mathbf{C}\left(\mathbf{A}^{\mathrm{T}}\mathbf{PA}\right)^{-1}\mathbf{A}^{\mathrm{T}}\mathbf{Ph}_i}{\mathbf{h}_i{}^{\mathrm{T}}\mathbf{PQ}_{\mathbf{v}}\mathbf{Ph}_i}}\,\sigma_0\delta_0 . \tag{10}$$

Since there is a Protection Level corresponding with each measurement and it is desired to obtain a single Protection Level for the position, the largest PL_i is conservatively selected as the position's Protection Level. Even if the position passes outlier testing, it is still considered unreliable for positioning if the Protection Level is greater than the Alert Limit. It is often found that this occurs when there is poor geometry and there is a lack of redundant measurements.

2.2 The New MDB Procedure

From Eqs. (2) and (10) it can be seen that the threshold for the outlier statistic and the Protection Level are dependent on the set Type I error probability. However, the Type I error probabilities can be made dependent on the Alert Limit by setting the i^{th} Protection Level equal to the Alert Limit, in Eq. (10), which yields

$$\delta_i = \frac{\mathrm{AL}\sqrt{\mathbf{h}_i^{\mathrm{T}}\mathbf{P}\mathbf{Q}_{\mathbf{v}}\mathbf{P}\mathbf{h}_i}}{\sigma_0\sqrt{\mathbf{h}_i^{\mathrm{T}}\mathbf{P}\mathbf{A}\left(\mathbf{A}^{\mathrm{T}}\mathbf{P}\mathbf{A}\right)^{-1}\mathbf{C}^{\mathrm{T}}\mathbf{C}\left(\mathbf{A}^{\mathrm{T}}\mathbf{P}\mathbf{A}\right)^{-1}\mathbf{A}^{\mathrm{T}}\mathbf{P}\mathbf{h}_i}}\,. \quad (11)$$

Then, with Eq. (4) the i^{th} Type I error probability, to be employed with the i^{th} outlier statistic, can be obtained as

$$\alpha_i = 1 - \mathrm{F}((\chi^2_{\beta_i,1,\delta_i^2}), 1) \quad (12)$$

where F(x, v) is the cumulative distribution function of a chi-squared distribution with v degrees of freedom. If all the outlier statistics pass with respect to their α_i values, then it is concluded that the position has sufficient integrity. Otherwise, the measurement that corresponds with the largest outlier statistic is rejected and the α_i values are updated. This is continued until all the outlier statistics pass, or there are an insufficient number of measurements remaining.

Whilst the position has integrity when all of the outlier statistics pass, the position may not have sufficient continuity for the particular application. Nevertheless, the Probability of a False Alert, and the continuity probability, can be estimated via (Sĭdák, 1968)

$$\mathrm{P}_{\mathrm{FA}} \leq 1 - \prod_{i=1}^{n}(1-\alpha_i)\,. \quad (13)$$

If the computed continuity probability is insufficient for the particular application, then the position is considered unreliable on the grounds of continuity.

2.3 Comparing the MDB Procedures

To demonstrate the benefits of the new FDE procedure compared to the conventional FDE procedure, the i^{th} Protection Level was initially plotted against the Type I error probability, in Fig. 2, to illustrate the reductions in the Protection Level that can be achieved by simply changing the Type I error probability.

From Fig. 2 it can be seen that the i^{th} Protection Level can be reduced to zero by simply increasing the Type I error probability. However, the most pronounced reduction occurs when α is less than 10% and as α approaches 1-β. In addition, the larger the β probability initially set the greater the reduction in the Protection Level that can be achieved with a small α value.

Further comparisons of the conventional and new FDE procedures were also carried out by applying the procedures to the 24 hours of GPS data shown in Fig. 3, which contains between 6 and 11 satellites and has DOP values that are less than five. In addition, the Probability of a Missed Detection was set to 20%, the Probability of a False Alert was set to 1%, and the Horizontal and Vertical Alert Limits were set to 25m and 50m respectively.

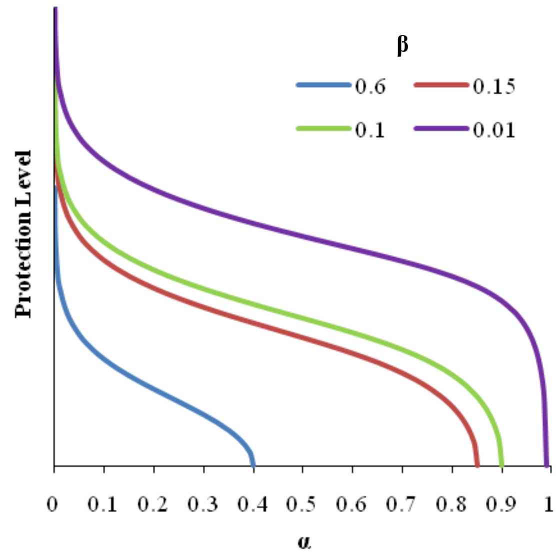

Figure 2: The Protection Level as α Increases for a given β

If it is initially considered that there are no continuity requirements, and that it is simply desired to obtain a position with integrity, then in the conventional MDB procedure based on the assumption of a single bias it was found that all the positions pass statistical testing, with Eq. (2). Therefore, the positions can be determined to have sufficient integrity solely based on the comparisons of the Protection Levels, from Eq. (10), and Alert Limits that are shown in Figs. 4 and 6. When the new MDB procedure was employed, it was found that 99% of the positions successfully pass outlier testing with respect to the Horizontal and Vertical Alert Limits. The percentage of time that a position with horizontal and vertical integrity was obtained is summarised in the first row of Table 1.

Comparatively, it can be seen from Table 1 that the new MDB procedure produces a significant increase in the percentage of time that a position with integrity is obtained. The reason for this can be explained with the assistance of Figs. 5 and 7, which plot the horizontal and vertical Probability of a False Alert for the new MDB procedure. When the Protection Level in the conventional procedure is greater than the Alert Limit, the new procedure increases the Type I error probabilities and the Probability of a False Alert. Since the Type I error probabilities and the Probability of a False Alert are often less than one, there is still a reasonable chance of all the outlier statistics passing. As a result, a position with the set Probability of a Missed

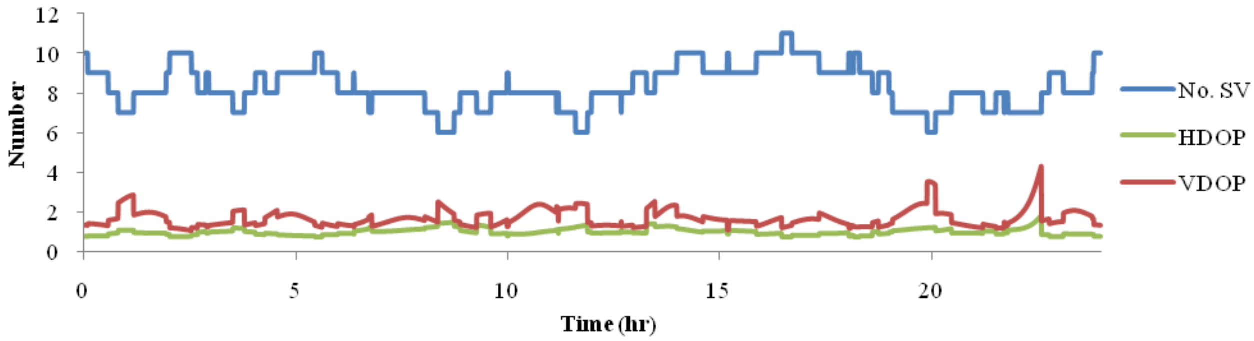

Figure 3: Number of Satellites and DOPs

Table 1: Availability Rates of the Conventional and New MDB Procedures

Availability	**Horizontal**		**Vertical**	
MDB Procedure	**Conventional**	**New**	**Conventional**	**New**
No Continuity Requirement	55%	99%	66%	99%
Continuity Requirement	55%	74%	66%	76%

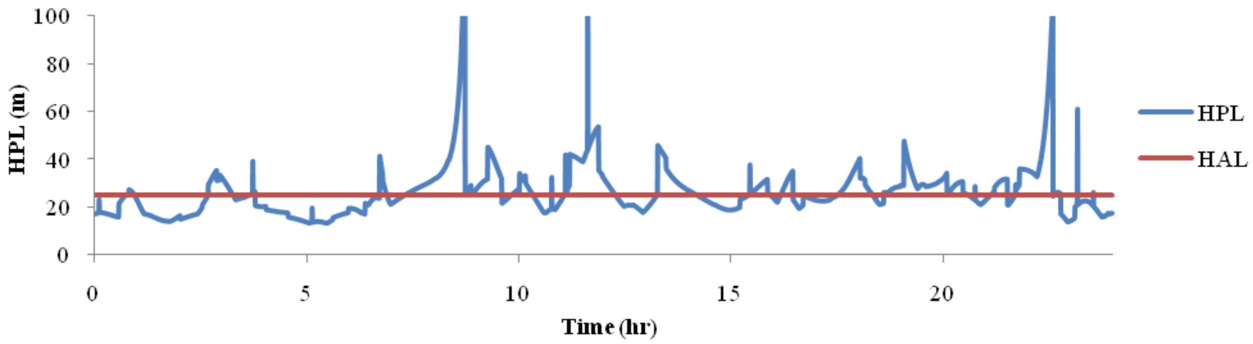

Figure 4: Horizontal Protection Level Based On the Conventional MDB Procedure

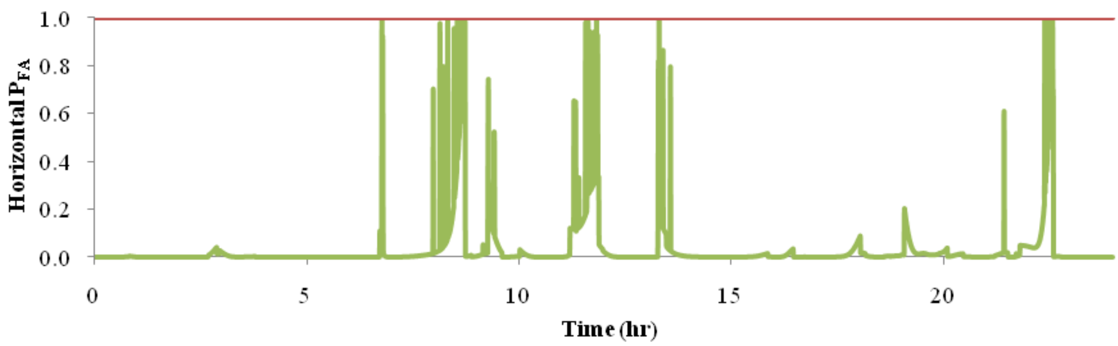

Figure 5: The New MDB Procedure's Horizontal Probability of a False Alert

Detection is more often obtained with the new MDB procedure.

Another interesting scenario is to consider the case where there is also a continuity requirement to be satisfied. If the continuity requirement in this case results in a required False Alert Probability of 1%, then the conventional MDB procedure results in the same availability rates as before. However, in the new MDB procedure 1% of the positions fail outlier testing, with respect to the Horizontal and Vertical Alert Limits, which results in the Probability of a False Alert being 100%. Therefore, a position can be determined to have sufficient integrity and continuity solely from a comparison of the computed and required continuity in

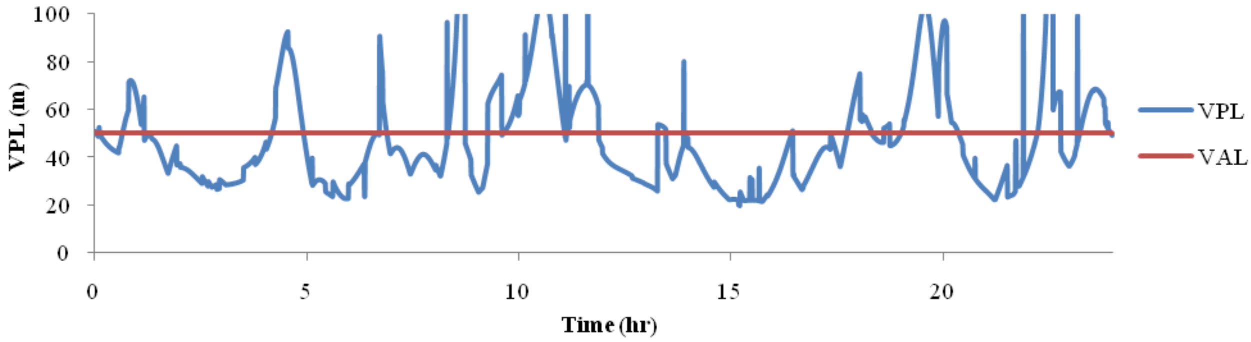

Figure 6: Vertical Protection Level Based On the Conventional MDB Procedure

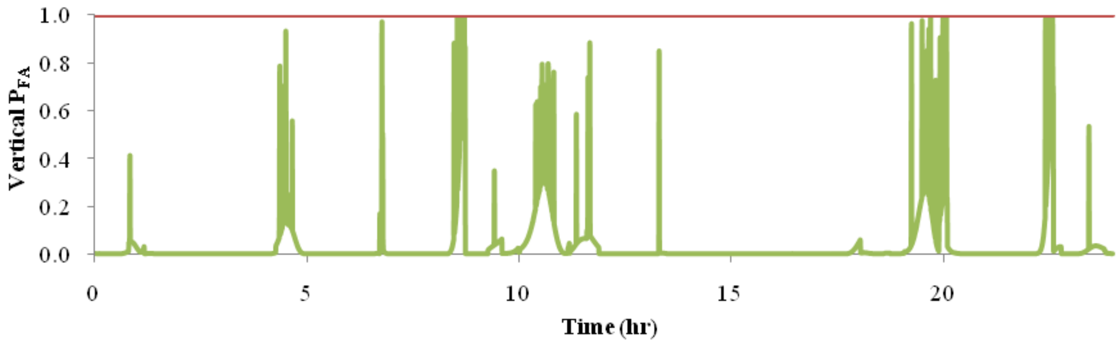

Figure 7: The New MDB Procedure's Vertical Probability of a False Alert

Figs. 5 and 7. The results of this comparison are also summarised in Table 1.

Once again, from Table 1 it can be seen that there is a significant increase in the availability rates with the new MDB procedure compared to the conventional MDB procedure. Whilst this may appear to be a surprising result at first, it can be explained by the conventional procedure setting the same Type I error probabilities, for each of the outlier tests, and conservatively selecting the largest of the Protection Levels as the position's Protection Level. However, the new MDB procedure removes this conservative approximation by setting all the Protection Levels equal to the Alert Limit, which on average reduces the Type I error probabilities. Thus, a smaller False Alert Probability is obtained which results in the increased rates of availability.

3. Fault Detection and Exclusion For Two Biases

In determining if a position has sufficient integrity, it is common to assume that there is at most a single bias. Hence, it is from this perspective that the preceding MDB method has been derived. However, it has been demonstrated by Knight et al. (2009) that in the presence of two or more biases the theories based on the assumption of a single bias are incapable of providing a position with the set level of integrity. Therefore, in cases where there is a high probability of multiple biases occurring it may be deemed necessary to provide integrity based on the assumption of two biases.

3.1 The Conventional MDB Procedure

If two biases are considered, then in the conventional MDB procedure the single outlier test is replaced with the outlier test for two outliers, which is given by (Cook and Weisberg, 1982; Förstner, 1983)

$$w^2 = \frac{\ell^{\mathrm{T}}\mathbf{PQ}_{\mathrm{v}}\mathbf{PH}(\mathbf{H}^{\mathrm{T}}\mathbf{PQ}_{\mathrm{v}}\mathbf{PH})^{-1}\mathbf{H}^{\mathrm{T}}\mathbf{PQ}_{\mathrm{v}}\mathbf{P}\ell}{\sigma_0^{\;2}} \sim \chi^2_{1-\alpha,\,2} \tag{14}$$

where $\mathbf{H}$ is the n by two matrix

$$\mathbf{H} = [\,\mathbf{h}_i \;\; \mathbf{h}_j\,]\,. \tag{15}$$

Since there are $\binom{n}{2}$ combinations of the $\mathbf{H}$ matrix that can be formed, then there is also an equal number of outlier statistics and associated Type I error probabilities. Therefore, with the continuity probability the Type I error probabilities of the outlier tests can be obtained with (Dykstra, 1980)

$$\alpha = 1 - \sqrt[\binom{n}{2}]{1 - \mathrm{P_{FA}}}\;. \tag{16}$$

If one or more of the outlier statistics fails, then it is deduced that there are one or more biases within the measurements. In the case of FDE, the measurements that correspond with the largest outlier statistic are rejected and the outlier tests are reapplied. This is continued until all the outlier statistics pass or there are an insufficient number of measurements remaining.

Like the single outlier case, even if all of the outlier statistics pass there is still a possibility of multiple biases going undetected that cause the positioning solution to be in error greater than the Alert Limit. Therefore, the MDB method is used to indicate the region in which the Probability of a Missed Detection is unacceptable. Hence, setting the Type II error probability of the outlier test equal to the Probability of a Missed Detection the corresponding shift in the outlier statistic can be obtained as (Knight et al., 2009)

$$\chi^2_{1-\alpha,2} = \chi^2_{\beta,2,\delta_0'} . \tag{17}$$

In the presences of two biases though the expected shift in the outlier statistic is given by

$$\delta' = \frac{\nabla\mathbf{s}^{\mathrm{T}}\mathbf{H}^{\mathrm{T}}\mathbf{P}\mathbf{Q}_{\mathbf{v}}\mathbf{P}\mathbf{H}\nabla\mathbf{s}}{\sigma_0{}^2} \tag{18}$$

where $\mathbf{H}$ corresponds with the bias vector, $\nabla\mathbf{s}$. Therefore, substituting δ_0' the MDB vector is obtained as (Förstner, 1983; Knight et al., 2009)

$$\delta_0' = \frac{\nabla_0\mathbf{s}^{\mathrm{T}}\mathbf{H}^{\mathrm{T}}\mathbf{P}\mathbf{Q}_{\mathbf{v}}\mathbf{P}\mathbf{H}\nabla_0\mathbf{s}}{\sigma_0{}^2} , \tag{19}$$

which defines a range of MDB vectors that correspond with the $\mathbf{H}$ matrix. To determine the impact of the MDBs on the positioning solution, it is initially required to notice that the expected shift in the least squares solution caused by two biases is given by

$$\nabla\mathbf{x} = (\mathbf{A}^{\mathrm{T}}\mathbf{P}\mathbf{A})^{-1}\mathbf{A}^{\mathrm{T}}\mathbf{P}\mathbf{H}\nabla\mathbf{s} . \tag{20}$$

Therefore, substituting the MDB vector, and removing the parameters of interest, the Protection Level for a given $\mathbf{H}$ matrix can be obtained as

$$\mathrm{PL} = \nabla_0\mathbf{s}^{\mathrm{T}}\mathbf{H}^{\mathrm{T}}\mathbf{P}\mathbf{A}\left(\mathbf{A}^{\mathrm{T}}\mathbf{P}\mathbf{A}\right)^{-1}\mathbf{C}^{\mathrm{T}}\mathbf{C}\left(\mathbf{A}^{\mathrm{T}}\mathbf{P}\mathbf{A}\right)^{-1}\mathbf{A}^{\mathrm{T}}\mathbf{P}\mathbf{H}\nabla_0\mathbf{s} . \tag{21}$$

Due to there being a range of MDB vectors for a given $\mathbf{H}$ matrix, there is also a range of Protection Levels. Since it is desired to place an upper bound on the region in which the Probability of a Missed Detection is unacceptable, Angus (2006) and Knight et al. (2009) define the Protection Level, for a given $\mathbf{H}$ matrix, as the maximum Protection Level obtainable subject to the constraint of Eq. (19). Therefore, using Rayleigh-Ritz quotient the maximum Protection Level is obtained as

$$\mathrm{PL}_{\mathrm{Max}} = \sigma_0\sqrt{\delta_0'\lambda_{\mathrm{Max}}} \tag{22}$$

where λ_{Max} is the maximum eigenvalue of

$$\begin{aligned}(\mathbf{H}^{\mathrm{T}}\mathbf{P}\mathbf{Q}_{\mathbf{v}}\mathbf{P}\mathbf{H})^{-1}\mathbf{H}^{\mathrm{T}}\mathbf{P}\mathbf{A}(\mathbf{A}^{\mathrm{T}}\mathbf{P}\mathbf{A})^{-1}\mathbf{C}^{\mathrm{T}} \\ \times\mathbf{C}(\mathbf{A}^{\mathrm{T}}\mathbf{P}\mathbf{A})^{-1}\mathbf{A}^{\mathrm{T}}\mathbf{P}\mathbf{H}\mathbf{v} = \lambda\mathbf{v}\end{aligned} . \tag{23}$$

Since it is desired to obtain a single Protection Level for the position, the largest of the $\mathrm{PL}_{\mathrm{Max}}$ values is conservatively selected as the position's Protection Level. Therefore, the position is still considered unreliable if the position's Protection Level is greater than the Alert Limit.

3.2 The New MDB Procedure

When multiple outliers are considered, it can likewise be seen that the threshold for the outlier test and the Protection Level are dependent on the Type I error probability. Therefore, setting the Protection Level equal to the Alert Limit, in Eq. (22), yields

$$\delta_0' = \frac{(\mathrm{AL})^2}{\sigma_0{}^2\lambda_{\mathrm{Max}}} \tag{24}$$

from which the Type I error probability is obtained as

$$\alpha = 1 - \mathrm{F}((\chi^2_{\beta,2,\delta_0'}), 2) . \tag{25}$$

By employing this Type I error probability in the outlier test, it can be determined that the position has integrity when all the outlier tests pass. However, in the case where one or more of the outlier statistics fails the measurements corresponding with the largest outlier statistic are rejected and the outlier test is reapplied.

If the particular application also has continuity requirements, then the Probability of a False Alert, and the continuity probability, can also be estimated via (Dykstra, 1980)

$$\mathrm{P}_{\mathrm{FA}} \leq 1 - \prod_{k=1}^{\binom{n}{2}}(1-\alpha_k) . \tag{26}$$

3.3 Comparing the MDB Procedures

To compare the conventional MDB procedure with the new MDB procedure, when two biases are considered, the Protection Level was initially plotted against the Type I error probability for a range of β values, in Fig. 8.

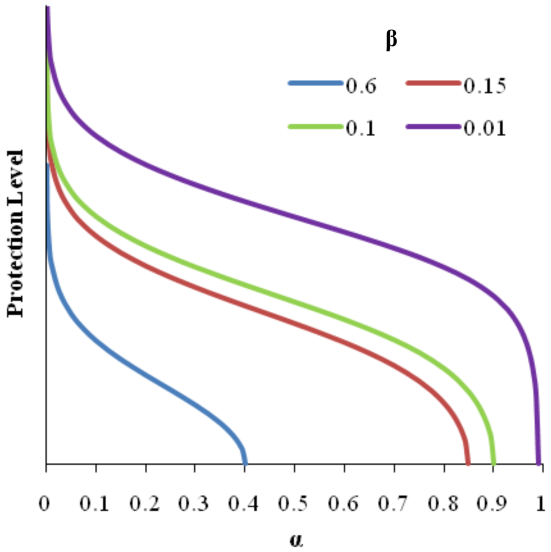

Figure 8: The Protection Level as α Increases for a given β

From Fig. 8 it can be seen that when two biases are considered, a significant reduction in the Protection Level can be achieved by increasing the Type I error probability. Further, comparing with the single outlier case in Fig. 2 shows that the only difference is that the Protection Level is slightly larger for the same α and β values.

The conventional and new MDB procedures were also applied to the 24 hours of GPS data shown in Fig. 3, but considering that there are two biases. The resulting Protection Levels are displayed in Figs. 9 and 11, and the False Alert Probabilities are displayed in Figs. 10 and 12.

From Fig. 9 it can be seen that horizontally the conventional MDB procedure is unable to provide a position with integrity. In the vertical sense, only a small percentage of the positions have Protection Levels that are less than the Alert Limit. In addition, these positions must also pass outlier testing. Comparing with the single outlier case, it is clear that the more biases considered the larger the Protection Levels become, which agrees with the finding of Knight et al. (2009; 2010).

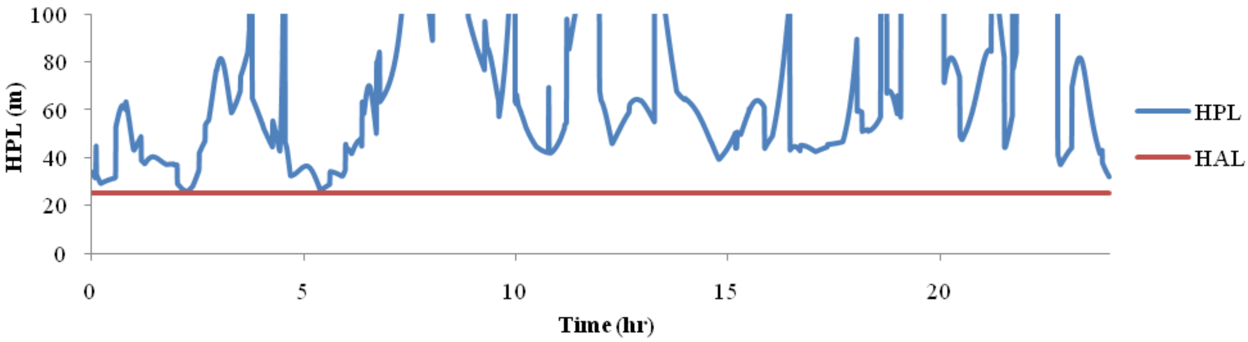

Figure 9: Horizontal Protection Level Based On the Conventional MDB Procedure

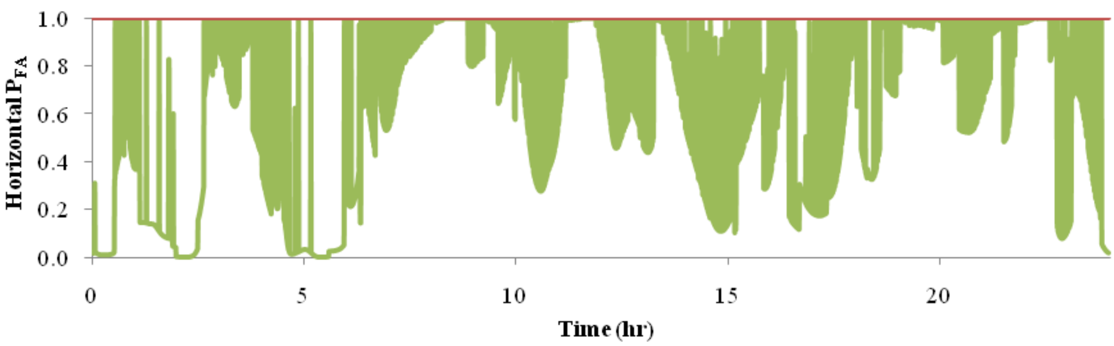

Figure 10: The New MDB Procedure’s Horizontal Probability of a False Alert

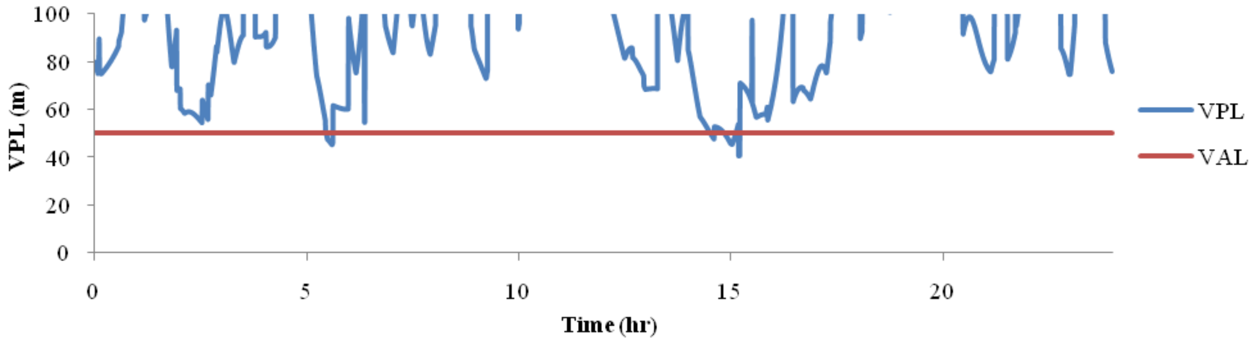

Figure 11: Vertical Protection Level Based On the Conventional MDB Procedure

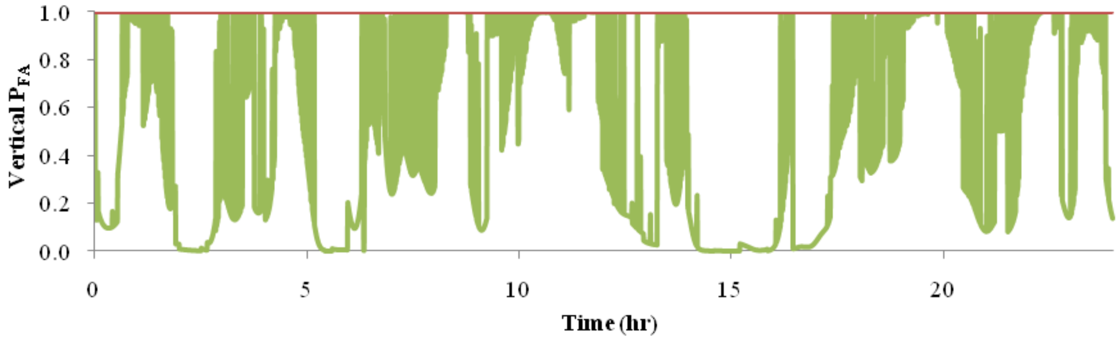

Figure 12: The New MDB Procedure's Vertical Probability of a False Alert

Table 2: Availability Rates of the Conventional and New MDB Procedures

Availability	**Horizontal**		**Vertical**	
MDB Procedure	**Conventional**	**New**	**Conventional**	**New**
No Continuity Requirement	0%	87%	2%	88%
Continuity Requirement	0%	3%	2%	11%

If the new MDB procedure is considered, then it is found that for a large percentage of the time the False Alert Probability is equal to one, which means that the position failed to pass outlier testing. When continuity requirements are also taken into account, then there is only a small percentage of the time that the position has integrity and continuity. The reason for this can be explained by the larger Protection Levels, which require the Type I error probability, and the Probability of a False Alert, to be further increased in order to set the Protection Levels equal to the Alert Limit.

From the final availability rates, with and without continuity requirements, that are summarised in Table 2 it can be seen that the new MDB procedure more frequently provides a satisfactory position.

4. Concluding Remarks

Providing integrity with the conventional MDB procedure requires the Type I error probability of the outlier tests to be preset. Traditionally, the Type I error probability has been set to avoid the unnecessary rejection of measurements or based on the continuity requirements. This paper proposes a new procedure of setting the Type I error probability, based on the integrity requirements, by initially setting the Protection Levels equal to the Alert Limit.

Comparisons of the new MDB procedure with the conventional MDB procedure have shown that the new procedure more often provides a position with integrity. This has been found to be irrespective of the continuity requirements and the number of biases considered.

The fundamental reason for the higher rates of availability achieved by the new procedure of applying the MDB method is that it removes many of the conservative assumptions that exist within the conventional procedure. These include the restriction of the continuity probability when the position's Protection Level is less than the Alert limit and the assumption that the biases always corresponds with the largest Protection Level.

Since the proposed procedure increases the Availability of a position with Integrity and is simple to apply, there appears to be few reasons as to why the proposed procedure should not be employed. The main reason though appears to be in the case where the measurements are remeasured since there is a higher probability of the measurements being rejected. However, in other applications, that may or may not have continuity requirements, the main reason appears to be that the new procedure is slightly more computationally intensive than the conventional approach. Even considering this, it appears that the new MDB procedure should still be used in preference to the conventional approach.

Even though the new MDB procedure increases the percentage of time that a position with integrity can be obtained, there are still times when a position with integrity cannot be obtained due to the very poor geometry. Since the new MDB procedure, removes many of the conservative assumptions that exist within the conventional procedure. It appears that to further increase the percentage of time that a position with integrity can be obtained the new MDB procedure must be extended to incorporate a dynamic model, via the use of Kalman filter, and/or increase the number of measurements, via sensor fusion.

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Biography

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