Journal of Global Positioning Systems (2011)
Vol.10, No.2 :136-148
DOI: 10.5081/jgps.10.2.136

Journal of Global Positioning Systems
2011

Investigation of Different Interpolation Models Used in Network-RTK for the Virtual Reference Station Technique

Al-Shaery, A.[1,2], Lim, S.[1], and Rizos, C.[1]
[1] *School of Surveying and Spatial Information System, the University of New South Wales, Sydney, NSW, Australia*
[2] *Civil Engineering Department, Umm Al-Qura University, Makkah, Saudi Arabia*

Abstract

This paper thoroughly investigates several approaches to implementing the GNSS network-based real-time positioning technique, which requires the estimation of atmospheric corrections on an epoch-by-epoch basis for RTK. In this study, a network of Continuously Operating Reference Stations in New South Wales, known as CORSnet-NSW, was utilised to: 1) obtain atmospheric residuals from each reference station, and 2) determine network correction for a rover operating in the area covered by the network using several interpolation methods. Applying the atmospheric corrections obtained by the interpolation methods, "synthetic" measurements at a virtual reference station are generated and then used for rover positioning. Field tests with various master-rover baseline lengths ranging from 21 to 62km indicate that a range of 1.9 to 6.5cm of horizontal positioning accuracy is achieved. In this study, the performance of geostatistical (Oridinary Kriging Method and Least Squares Collocation Method) and deterministic (Linear Combination Method, Linear Interpolation Method, Low-order Surface Method and Multiquadric Surface Fitting Method) interpolation methods used in GNSS network-based RTK positioning were also analysed in order to identify the optimal method for mitigating atmospheric effects for real-time kinematic applications under different network geometries.

Keywords: GNSS, VRS, interpolation methods.

1. Introduction

Due to the presence of atmospheric biases and orbit errors in between-receiver single-differenced observables, the performance of standard Real-Time Kinematic (RTK) may become degraded if the baseline length exceeds about 10km for GPS-based CPH relative positioning (Rizos, 2002, Wanninger, 1995). Many researchers have tested the use of multiple reference stations to overcome this baseline length limitation. The network-RTK (N-RTK) approach provides a solution that can overcome this constraint by estimating correction terms for atmospheric biases and orbit errors for a user receiver within the coverage area of the reference station network. This is carried out in three steps (Al-Shaery et al., 2010a, Chen et al., 2000, Vollath et al., 2000, Yi and Grejner-Brzezinska, 2003) :

- Integer ambiguities between base stations are first resolved.
- Secondly, atmospheric errors (delays) in the GPS measurement are estimated (as the coordinates of the reference stations are known). At this stage, the atmospheric delays over the network are also modelled.
- Thirdly, the delay corresponding to a user location is interpolated either at network server (if the server-based positioning approach is used) or at the user receiver (standard RTK mode).

There are two issues in NRTK. The first issue is the precise estimation of the distance-dependent errors affecting GNSS signals at the reference stations. The second issue is the accuracy of interpolation methods of the corrections for these errors for a user receiver located inside the network. The interpolation algorithms may work even if a rover must be placed outside the network. However, in such a case, the algorithms actually behave as extrapolators and therefore the positioning accuracy may decline. This paper will focus on the second issue. Details on the first issue can be found in, for example, Gao et al. (1997), Colombo et al. (1999), Chen et al. (2000), Dai et al. (2001) and Zhang et al. (2009).

Cannon and Fotopoulos (2001) and Dai et al. (2003) investigated the advantages and disadvantages of various interpolation methods. These methods included the Linear Combination Model (Han and Rizos, 1996), the Distance-based Linear Interpolation Method (Gao et al., 1997), the Linear Interpolation Method (Wanninger, 1995, Wubbena et al., 1996), the Low-order Surface Model (Fotopoulos and Cannon, 2000, Wubbena et al., 1996), and the Least Squares Collocation Method (Raquet, 1997, Van der Marel, 1998).

It is known that the ionosphere modelling for NRTK positioning based on a spherical harmonic expansion is not effective, consequently, interpolation methods can be used to provide similar or better results (Gao and Liu, 2002, Wielgosz et al., 2003).

The use of Kriging and Multiquadric methods for ionosphere mapping were investigated by (Wielgosz et al., 2003). Geisler (2006) examined the impact of different interpolation methods (Distance-based Linear Interpolation Method, Low-order 2D Surface Model and the Least Squares Collocation Method) and the selection of reference stations on user positioning in NRTK using the Master Auxiliary Concept (MAC). Wu (2009) examined and tested three interpolation models (the Linear Interpolation Method, the Distance-based Linear Interpolation Method and the Low-order Surface Model) to identify the best method for NRTK accuracy using the GPS CORS network in Victoria, Australia.

Interpolation methods can be divided into two main groups: geostatistical and deterministic. Geostatistical methods use statistical properties of measured points, whereas deterministic methods use predefined mathematical functions and calculate the function's coefficients from measured points. Several attempts have been made to compare different interpolation methods. However, as far as the authors are aware no comprehensive comparison between geostatistical and deterministic methods has been reported in the literature by employing the same spatial and temporal data i.e. the same study area, the same network, and the same observation session. Moreover, the impact of rover location, whether it is inside or outside the network coverage, on the performance of the interpolation methods has not been thoroughly investigated.

This paper reviews the performance of geostatistical (Ordinary Kriging Method and Least Squares Collocation Method) and deterministic (Linear Combination Method, Linear Interpolation Method, Low-order Surface Method and Multiquadric Surface Fitting) interpolation methods used for modelling the atmospheric delay errors (ionosphere and troposphere) from a CORS network for GPS RTK rover positioning under different network geometries. First, reference stations are used to estimate atmospheric residuals. Secondly, the interpolation methods are used to interpolate atmospheric corrections for the location of a user receiver. Thirdly, a "virtual reference station" (VRS) is established using the interpolated correction. Fourthly, RTK positioning between the VRS and the user receiver is carried out. Finally, an accuracy assessment is performed to evaluate the performance of the examined methods.

2. Methodology

The main objective of this paper is to compare the performance of kinematic GPS positioning of several interpolation methods. Several modules were developed for this study (see Figure 1).

2.1 CORS-Network module

This module estimates atmospheric residuals at each GPS reference station. Using a network of reference stations with precisely known positions, atmospheric residuals to every observed satellite can be obtained. Determining network corrections is carried out using software developed by the School of Surveying and Spatial Information Systems (SSIS) at the University of New South Wales (UNSW).

A total of *n-1* single-differenced (between-receivers) ionospheric and tropospheric residuals are obtained from this module. Such residuals are later used to form VRS observables in the VRS module. More details concerning this module are given in (Zhang et al., 2009).

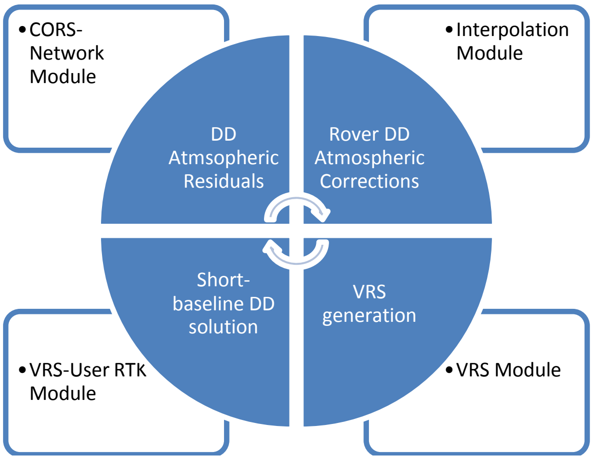

Figure 1: Test Modules.

2.2 Interpolation Module

This module is used to generate corrections for user receivers within the network. Several interpolation methods have been developed over the past few years. The most widely used interpolation methods in NRTK were tested in this research. These methods can be grouped into two main classes:

- Geostatistical (Ordinary Kriging-OKR and Least Squares Collocation Method –LSC)
- Deterministic (Linear Combination Method-LCM, Linear Interpolation Method-LIM, Low-order Surface Method-LSM and Multiquadric Surface Fitting-MSF)

Ordinary Kriging (OKR)

OKR is used to estimate the value at a location using neighbouring sample of data whose semi-variogram is known (Wackernagel, 2003). The semi-variogram provides information for interpolation sampling optimisation and for determining spatial patterns. Compared to other interpolation methods such as the inverse distance method, Kriging is a geostatistical method that takes into account the spatial and temporal correlation of data sources using the semi-variogram of the sample data (Wielgosz et al., 2003). Furthermore, unlike these deterministic interpolation methods, this geostatistical method provides an indication of the error in the form of a variance (Wackernagel, 2003). Kriging variance is given by:

$$\hat{\sigma}_e^2 = \sum_{i=1}^{n} \lambda_i \, \gamma(x_i, x_0) + \phi \quad (1)$$

With $\sum_{i=1}^{n} \lambda_i = 1$

where

- λ_i is the Kriging weight parameter for each station (sample point) involved in the Kriging interpolation
- ϕ is the Lagrange multiplier (LM)
- $\gamma(x_i, x_0)$ is the semi-variance between the user receiver (x_0)and reference stations (x_i)

The Kriging method is carried out in three steps (Al-Shaery et al., 2010b):

1. Constructing an experimental variogram.
2. Fitting the experimental variogram to an appropriate theoretical model.
3. Determining the weight parameters for each reference station.

In the second step, the optimum parameters that fit the experimental variogram to an appropriate theoretical variogram model are determined. A validation technique known as cross-validation is implemented to assist the selection of the appropriate theoretical model. Based on this step, the appropriate model is used in the next step to determine the Kriging weight parameters (λ_i) for each reference station and the LM (ϕ). Three functional models were tested: 1) a spherical model, 2) an exponential model, and 3) a Gaussian model. It has been found that the last two models well fit the experimental variograms for both ionospheric and tropospheric residuals (Al-Shaery et al., 2010a). It was concluded in (Al-Shaery et al., 2010a, Al-Shaery et al., 2010b) that the exponential model is better able to model spatially-correlated errors, such ionospheric delays. Hence in this paper the exponential model was used.

The final step requires the computation of the weight parameters for each reference station:

$$A \cdot \alpha = b \quad (2)$$

where

$$A = \begin{bmatrix} \gamma_{1,1} & \gamma_{1,2} & \cdots & \gamma_{1,n-1} & 1 \\ \gamma_{2,1} & \gamma_{2,2} & \cdots & \gamma_{2,n-1} & 1 \\ \vdots & \vdots & \vdots & \vdots & 1 \\ \gamma_{n-1,1} & \gamma_{n-1,2} & \cdots & \gamma_{n-1,n-1} & 1 \\ 1 & 1 & \cdots & 1 & 0 \end{bmatrix}$$

$$\alpha = \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \vdots \\ \lambda_{n-1} \\ \phi \\ \square \end{bmatrix} \qquad b = \begin{bmatrix} \gamma_{u,1} \\ \gamma_{u,2} \\ \vdots \\ \gamma_{u,n-1} \\ 1 \end{bmatrix}$$

The interpolation coefficients can be determined by:

$$\vec{\alpha} = (\mathrm{A^T A})^{-1} A^T b$$

The matrix A represents the semi-variances of reference stations. The variance between the user receiver and reference stations are calculated to form the vector b, based on a selected theoretical variogram. α is the weighted parameters corresponding to each baseline. Here the following exponential variogram was applied:

$$\gamma(\hbar_{i,j}) = c_0 + c_1 \left\{1 - exp\left(-\frac{\hbar_{i,j}}{a}\right)\right\} \quad (3)$$

The lags ($\hbar_{i,j}$) represent the distance between reference stations i and j, c_0, c_1 and a are the model parameters that fit the experimental variogram to the exponential model (Al-Shaery et al., 2010a). Applying the weight parameters to the atmospheric residuals of each baseline formed from the master station to a reference station, the corresponding correction for the user location can be determined by:

$$\hat{v}_{un}^s = \sum_{i=1}^{n-1} \alpha_i v_i^s \quad (4)$$

where

- $\hat{v}_{un}^s$ is the residual of the baseline from the user to the master station.
- v_i^s is the residual of i-th baseline from the master to the i-th reference station.

Linear Combination Method (LCM)

LCM was developed by Han and Rizos (1996, 1998) to model distance-dependent biases such as orbit ($\Delta\rho_{orb,i}$), ionospheric ($\Delta d_{ion,i}$) and tropospheric biases ($\Delta d_{trop,i}$), and to mitigate multipath ($\Delta d_{mp,i}$) and noise ($\varepsilon_{\sum_{i=1}^{n} \alpha_i \cdot \Delta\varphi_i}$). The model is formed from a linear combination of single-differenced observations from a number of reference stations (n):

$$\sum_{i=1}^{n} \alpha_i \cdot \Delta\varphi_i = \sum_{i=1}^{n} \alpha_i \cdot \Delta\rho_i + \sum_{i=1}^{n} \alpha_i \cdot \Delta d\rho_i - c \cdot \sum_{i=1}^{n} \alpha_i \cdot \Delta dT_i + \lambda \cdot \sum_{i=1}^{n} \alpha_i \cdot \Delta N_i - \sum_{i=1}^{n} \alpha_i \cdot \Delta d_{ion,i} + \sum_{i=1}^{n} \alpha_i \cdot \Delta d_{trop,i} + \sum_{i=1}^{n} \alpha_i \cdot \Delta d_{mp,i}^{\varphi} + \varepsilon_{\sum_{i=1}^{n} \alpha_i \cdot \Delta\varphi_i} \quad (5)$$

Han and Rizos (1996, 1998) introduced the following conditions to determine the weight parameters (α_i):

$$\sum_{i=1}^{n} \alpha_i = 1 \quad (6)$$

$$\sum_{i=1}^{n} \alpha_i (\hat{X}_u - \hat{X}_i) = 0 \quad (7)$$

$$\sum_{i=1}^{n} \alpha_i^2 = Min \quad (8)$$

where

$\hat{X}_u$ is the horizontal coordinate vector of the user location.

$\hat{X}_i$ is the horizontal coordinate vector of the i-th reference station.

The modified general linear formula given below is based on the conditions given in Han and Rizos (1996):

$$\begin{bmatrix} 1 & 1 & \dots & 1 \\ \Delta X_{1n} & \Delta X_{2n} & \dots & \Delta X_{n-1,n} \\ \Delta Y_{1n} & \Delta Y_{2n} & \dots & \Delta Y_{n-1,n} \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_{n-1} \end{bmatrix} = \begin{bmatrix} 1 \\ \Delta X_{un} \\ \Delta Y_{un} \end{bmatrix} \quad (9)$$

This formula can be written in the matrix form:

$$A\alpha = b$$

where

$$\alpha = \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_{n-1} \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 1 & \dots & 1 \\ \Delta X_{1n} & \Delta X_{2n} & \dots & \Delta X_{n-1,n} \\ \Delta Y_{1n} & \Delta Y_{2n} & \dots & \Delta Y_{n-1,n} \end{bmatrix}$$

$$b = \begin{bmatrix} 1 \\ \Delta X_{un} \\ \Delta Y_{un} \end{bmatrix}$$

Where ΔX_{1n}, and ΔY_{1n} are the differences of the plane coordinate between the master reference station (n) and each reference station (2,..,n-1). α is (n-1) weight parameters of all baselines between the master station and each reference stations. ΔX_{un} and ΔY_{un} are the vector differences of plane coordinates between the master and the user stations.

With

$$\sum_{i=1}^{n-1} \alpha_i^2 = min \quad (10)$$

the interpolation coefficients can be estimated:

$$\vec{\alpha} = A^T (A^T A)^{-1} b \quad (11)$$

Applying this to the VRS NRTK approach, the user correction can be obtained as follows:

$$\hat{v}_v^s = [\alpha_1 \quad \alpha_2 \quad \dots \quad \alpha_{n-1}] \begin{bmatrix} v_{1n}^s \\ v_{2n}^s \\ \vdots \\ v_{n-1,n}^s \end{bmatrix} \quad (12)$$

Linear Interpolation Method (LIM)

This method is widely reported in literature to model the distance-dependent biases such as the ionospheric and tropospheric delays, and orbit errors. Wanninger (1995) utilised the LIM to compute double-differenced ionospheric corrections for a user receiver surrounded by three reference GPS stations. The interpolation was computed on an epoch-by-epoch and satellite-by-satellite basis using the known coordinates of the reference stations and the approximate position of the user receiver. Wubbena et al. (1996), Chen et al. (2000) and Vollath et al. (2000) applied the same model to interpolate the distance-dependent errors. The basic formula is:

$$\begin{bmatrix} \Delta X_{1n} & \Delta Y_{1n} \\ \Delta X_{2n} & \Delta Y_{2n} \\ \vdots & \vdots \\ \Delta X_{n-1,n} & \Delta Y_{n-1,n} \end{bmatrix} \begin{bmatrix} \hat{a} \\ \hat{b} \end{bmatrix} = \begin{bmatrix} v_{1n}^s \\ v_{2n}^s \\ \vdots \\ v_{n-1,n}^s \end{bmatrix} \quad (13)$$

$$A \cdot X = V$$

$$\begin{bmatrix} \hat{a} \\ \hat{b} \end{bmatrix} = (A^T A)^{-1} A^T V \quad (14)$$

Where

ΔX_{1n} and ΔY_{1n} are the differences of plane coordinate between the master reference station (n) and each reference station (2,..,n-1).

$\hat{a}$ and $\hat{b}$ are network coefficients for ΔX and ΔY, respectively.

Hence, the corresponding correction for the user location can be obtained:

$$\hat{v}_{un}^s = \hat{a} \cdot \Delta X_{un} + \hat{b} \cdot \Delta Y_{un} \quad (15)$$

ΔX_{un} , ΔY_{un} are the differences of plane coordinates between the master station and the user location.

Least Squares Collocation Method (LSC)

LSC is a prediction method whereby the best estimates of attributes will be determined at certain points where no values are measured but some linear functionals and covariance of this attribute are known (Collier, 1988, Grgich et al., 2006, Krarup, 1970). One of the early uses of LSC was to predict the gravity anomalies at all points on the surface of the earth using measurements made at some locations (Krarup, 1970). A proposal for the use of LSC to interpolate the ionospheric and tropospheric biases at user locations using known values of such biases at multiple reference stations was discussed by several researchers (Alves, 2004, Dai et al., 2003, Fortez, 2002, Raquet, 1997, Raquet, 1998, Van der Marel, 1998).

Raquet (1997, 1998) treated the ionospheric and tropospheric biases as one entity. In the proposals of Van der Marel, Fortez, Dai and Alves, the ionospheric and tropospheric biases are interpolated separately. Both Raquet and Fortez assumed that the stochastic model for the ionospheric error is a function of the distance between network stations. Van der Marel, Alves and Dai assumed that the covariance of the ionospheric error is dependent on the separation of the ionospheric pierce points corresponding to the observables from two stations and from the same satellite. Defining an accurate stochastic model for this method is a challenge.

A single layer model for the ionosphere is illustrated in Figure 2. A practical interpolator suggested by Van der Marel (1998) and also used by Dai et al. (2003) for ionospheric and tropospheric biases is:

$$\hat{v}_v^s = \begin{bmatrix} C_{v,1}^s & C_{v,2}^s & \cdots & C_{v,n-1}^s \end{bmatrix} \cdot \begin{bmatrix} C_0 & C_{1,2}^s & \cdots & C_{1,n-1}^s \\ C_{2,1}^s & C_0 & \cdots & C_{2,n-1}^s \\ \vdots & \vdots & \ddots & \vdots \\ C_{n-1,1}^s & C_{n-1,2}^s & \cdots & C_0 \end{bmatrix}^{-1} \cdot \begin{bmatrix} v_1^s \\ v_2^s \\ \vdots \\ v_{n-1}^s \end{bmatrix} \quad (16)$$

where

$C_{1,2}^s$	is an appropriate covariance function between reference station (1) and (2) (Figure 2).
$C_{v,1}^s$	is the spatial covariance function of user and reference station (1).
$\hat{v}_v^s$	is the correction for the user location.

For ionospheric biases, the covariance function is (Dai et al., 2003):

$$C_{1,2}^s = l_{max} - l_{1,2}^s \quad (17)$$

where

l_{max}	is height of the ionospheric layer (a value of 300km was adopted (see Figure 2).
$l_{1,2}^s$	is the difference between the height of ionospheric pierce points of baseline receivers (1) and (2) to the same satellite (s) (see Figure 2).

This covariance implies that l_{max} is larger than the largest distance between network stations and their ionospheric points, which results in more weight being given to the biases of close stations.

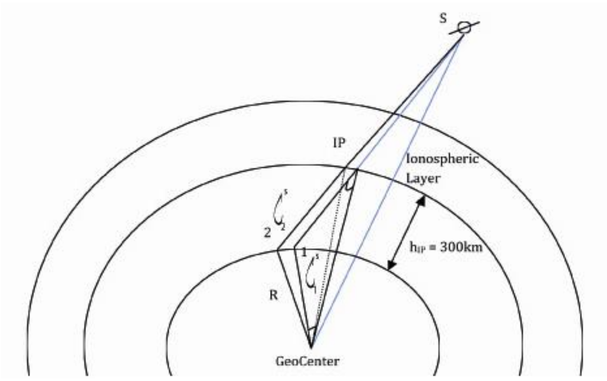

Figure 2: Single layer model for the ionosphere (not-to-scale).

For tropospheric biases, $C_{1,2}^s$ is a covariance function, which is linearly dependent on the distance between the stations (Cressie, 1993, Dai et al., 2003, Kitanidis, 1997, Van der Marel, 1998):

$$C(d_{ij}) = \begin{cases} 0 & d_{ij} = 0 \\ c \cdot \|d_{ij}\| & d_{ij} \neq 0 \end{cases} \quad (18)$$

where c is the slope of the variogram , $c = 1$. d_{ij} is the distance between station i and j.

Low-order Surface Method (LSM)

LSM can be also used to model spatially correlated biases which exhibit a high degree of spatial correlation (Cannon and Fotopoulos, 2001, Dai et al., 2003). The coefficients of the LSM can be determined using a least squares adjustment of data from the reference station network.

The LSM models can be 2D (horizontal coordinate differences, ΔX, ΔY) or 3D (horizontal, ΔX and ΔY, and height components, ΔH). For example, the following LSM function is a 2D model:

$$V = a \cdot \Delta X + b \cdot \Delta Y + c \quad (19)$$

This function can be used to model ionospheric or tropospheric residuals. If there is a significant difference in height between the reference stations, the following 3D equation can be used:

$$V=a\cdot\Delta X+ b\cdot\Delta Y+ c\cdot\Delta H+d \tag{20}$$

The coefficients of equation (20) can be obtained from reference stations' data by least squares:

$$\begin{bmatrix} \Delta X_{1n} & \Delta Y_{1n} & 1 \\ \Delta X_{2n} & \Delta Y_{2n} & 1 \\ \vdots & \vdots & \vdots \\ \Delta X_{n-1,n} & \Delta Y_{n-1,n} & 1 \end{bmatrix} \begin{bmatrix} \hat{a} \\ \hat{b} \\ \hat{c} \end{bmatrix} = \begin{bmatrix} \hat{v}_{1n}^{s} \\ \hat{v}_{2n}^{s} \\ \vdots \\ \hat{v}_{n-1,n}^{s} \end{bmatrix} \tag{21}$$

Its matrix form is:

$$A \cdot X = V$$

Its solution is:

$$\begin{bmatrix} \hat{a} \\ \hat{b} \\ \hat{c} \end{bmatrix} = (A^T A)^{-1} A^T V \tag{22}$$

Using the following, the residual for the user location can be obtained:

$$\hat{v}_{un}^{s} = \hat{a} \cdot \Delta X_{un} + \hat{b} \cdot \Delta Y_{un} + \hat{c} \tag{23}$$

Multiquadric Surface Fitting Method (MSF)

MSF estimates the interpolated value $v^*(x_0)$ as the sum of N individual quadric surfaces (Hardy, 1971, Shaw, 1994). Each quadric is positioned with its centre or vertex on each N surrounding data points. MSF is an efficient substitute to Kriging due to its light computation load (Hardy, 1984, Wielgosz et al., 2003). The general multiquadric equation from which the interpolated value can be obtained is:

$$v^*(x_0) = \sum_{j=1}^{n} c_j \, g_{0j} \tag{24}$$

Where

- c_j are the associated parameters defining the algebraic sign and flatness of the quadric terms.
- g_{0j} is a function of the distance between the user location and i-th reference stations.

The parameters (c_j) can be obtained as follows:

$$v(x_i) = \sum_{j=1}^{n} c_j \, g_{ij} \; (i = 1, \ldots, n) \tag{25}$$

where $v(x_i)$ is the i-th residual of i-th reference station.

Borga and Vizzaccaro (1997) investigated three quadric surfaces, hyperboloid surface of two sheets (Equation 26), paraboloid surface (Equation.27) and a conic surface (Equation.28).

$$g_{ij} = \left[(x_j - x_i)^2 + (y_j - y_i)^2 + a^2 \right]^{\frac{1}{2}} \tag{26}$$

$$g_{ij} = (x_j - x_i)^2 + (y_j - y_i)^2 \tag{27}$$

$$g_{ij} = \left[(x_j - x_i)^2 + (y_j - y_i)^2 \right]^{\frac{1}{2}} \tag{28}$$

Where

- a is about half the scale of the horizontal coordinates (Lee et al., 1974).
- x_j and x_i are plane coordinates of j-th and i-th reference station.

Lee et al. (1974) reported that hyperboloid surfaces may experience computational difficulties if a is set too high. He also found that paraboloid surfaces are not practical due to computational problems. Moore et al. (1989) suggested a modified equation for the general form which lacks the non-bias property (Borga and Vizzaccaro, 1997):

$$v^*(x_0) = \sum_{j=1}^{n} c_j \, g_{0j} + b \tag{29}$$

c and b are parameters that can be determined from the following system:

$$\begin{cases} v(x_i) = \sum_{j=1}^{n} c_j \, g_{ij} + b \quad (i = 1, \ldots, n) \\ \sum_{j=1}^{n} c_j = 0 \end{cases}$$

The conic surface, as recommended by Borga and Vizzaccaro (1997) is implemented here.

2.3 VRS module

This module generates VRS observables from the master station data (here the QUEN CORS station was selected as the master station, as shown in Figure 3) and a user pseudorange-based position. The VRS observables are constructed as follows (Hofmann-Wellenhof et al., 2008):

$$\varphi_r^s(X_v, t) = \varphi_r^s(X_m, t) + \frac{1}{\lambda}[\rho_r^s(X_v, t) - \rho_r^s(X_m, t)] + \frac{1}{\lambda}\Delta_r^s(X_v, t) \tag{30}$$

$$P_r^s(X_v, t) = P_r^s(X_m, t) + [\rho_r^s(X_v, t) - \rho_r^s(X_m, t)] + \Delta_r^s(X_v, t) \tag{31}$$

where:

- φ_r^s is the carrier-phase observable on the L1 or L2 carrier frequency (in cycles).
- $\varphi_r^s(X_v, t)$ is the carrier-phase observable of the VRS based on the code position of the user

receiver.

$P_r^s(X_v,t)$ is the code observable of the VRS based on the code position of the user receiver.

$\varphi_r^s(X_m,t)$ is the carrier-phase observable of the master station (QUEN station).

$P_r^s(X_m,t)$ is the code observable of the master station (QUEN station).

$\rho_r^s(X_v,t)$ is the geometric distance between satellite (s) and the receiver located at the VRS.

$\rho_r^s(X_m,t)$ is the geometric distance from a satellite to the master station receiver.

$\Delta_r^s(X_v,t)$ is the atmospheric (ionospheric and tropospheric) correction for the user position obtained from the Interpolation Module.

For the carrier-phase observable, the correction term is:

$$\Delta_r^s(X_v,t) = \Delta T_r^s(X_v,t) - \Delta I_r^s(X_v,t) \qquad (32)$$

For the code observable, the correction term is:

$$\Delta_r^s(X_v,t) = \Delta T_r^s(X_v,t) + \Delta I_r^s(X_v,t) \qquad (33)$$

It should be noted that the broadcast ephemeris is used to assess the suitability of the algorithm for RTK applications.

2.4 RTK module for VRS-User

In this module, a baseline solution is obtained for the short baseline between the rover user and the VRS. The Leica Geo Office (LGO) software, which is capable of processing kinematic GNSS data, was used for this task, processing the generated VRS RINEX file and the user receiver RINEX file in the kinematic baseline mode. The output was the estimate of the user position.

3. Experiment

Data from seven stations of the CORSnet-NSW network located in the Sydney region, Australia, were used to generate network corrections (Figure 3). The stations have a regular distribution with inter-station distance ranging from 20.7 to 62.5 km. The network algorithm (Zhang *et al.*, 2009) developed at the SSIS-UNSW was used to generate the network corrections in the form of between-receiver single-differences.

The data set used to determine the network corrections in this study was from 10 February 2009, and 9-hour data covering the period from 10:16:26am to 19:16:25pm (Sydney local time) was used to test and implement the proposed algorithm. This period includes an early afternoon when the ionospheric effect is expected to be high. Station QUEN (currently CHIP) was used as the master station for the calculation. Atmospheric residuals (ionospheric and tropospheric) between each reference station and the master station were computed for each satellite. The sample interval was 1 second.

Figure 3: Sydney Basin portion of CORSnet-NSW.

Three tests were carried out using the six single-differenced residuals estimated (QUEN-CWAN, QUEN-MGRV, QUEN-SPWD, QUEN-MENA QUEN-WFAL and QUEN-VLWD). The three network geometries for three tests are shown in Figures 4-6. In the first network geometry (netGeometry 1), VLWD was treated as the rover user. In this network geometry, the reference stations are equally spaced around the rover, which is a favourable reference-rover geometry in NRTK. Moreover, the rover is close to the master station (QUEN) (20.69km). In netGeometry 2 for the second test, station MGRV was used as the rover, which is located outside the network and not far away from the master station (44.83km). Station SPWD was selected as the rover in netGeometry 3 for the third test, which is located outside the network coverage and more distant from the master station (62.5km). Furthermore, the rover has higher ellipsoidal height (399.5m) compared to the rest of the network stations.

In each test five estimated single-differenced atmospheric residuals, excluding the rover-related set, were used in the Interpolation Module to estimate the corrections (QUEN-VLWD in the first test, QUEN-MGRV in the second, and QUEN-SPWD in the third). Then, the VRS Module established VRS observations based on the code-generated position which is close to the user location, and the master station's data (QUEN). The double-differenced solution for the short baseline was then obtained. As the coordinates of VLWD, MGRV and SPWD are already known, the performance of the each interpolation method can be directly assessed.

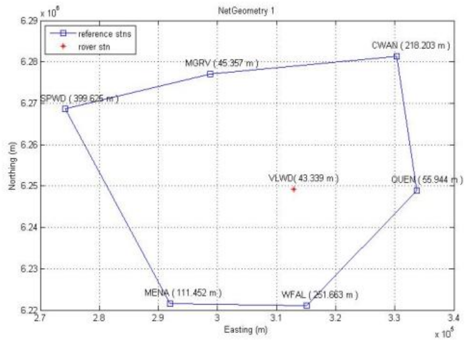

Figure 4: Network Geometry (netGeometry) 1 for test 1.

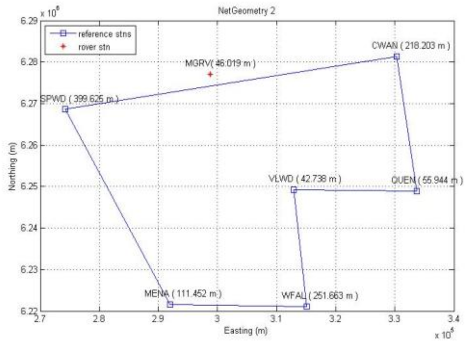

Figure 5: Network Geometry (netGeometry) 2 for test 2.

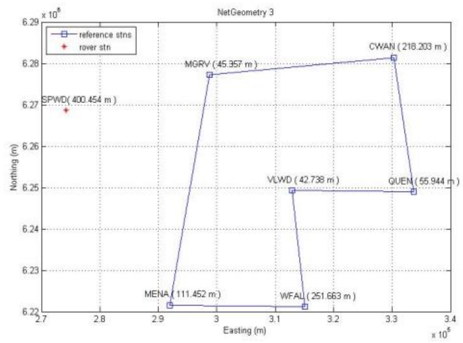

Figure 6: Network Geometry (netGeometry) 3 for test 3.

4. Results And Analyses

Based on the interpolated ionospheric and tropospheric corrections for the user location, the master station's (QUEN) observable and the code-derived position of the user receiver, a VRS observation RINEX file was constructed. The short baseline was processed using the LGO software.

VRS-VLWD. Table 1 lists the statistical values of difference between the estimated positions and the true coordinates of the rover station VLWD, and time series of the accuracy in the three directions are plotted in Figures 7-9 respectively. The success rate parameter represents the number of fixed solutions over the total number of epochs being processed.

Table 1: Statistical results for the VRS user's positioning (VLWD) (horizontal and vertical components).

	RMSE (m)		Success rate (%)
	Hz	Vt	
OKR	0.021	0.080	100
LCM	0.022	0.055	100
LIM	0.019	0.042	100
LSC	0.028	0.037	100
LSM	0.022	0.055	100
MSF	0.021	0.057	100

As can be seen from the graphs in Figures 7-9, a centimetre-level accuracy was achieved (though with lower accuracy in the vertical direction as expected). In addition to the fact that the GPS positioning accuracy in the height component is not as high as in the horizontal components, the effect of height differences between the reference stations and the user was not considered in these 2D interpolation methods. From Table 1, comparable accuracies of all the methods are noted from both parameters (the root mean square error and success rate). In terms of the horizontal positioning accuracy, the deterministic method LIM slightly outperformed the other methods while the geostatistical method LSC produced the lowest accuracy (2.8cm).

In terms of the vertical component, the performance of these methods was the reversed as the highest accuracy is LSC (3.8cm) and the lowest is OKR (8cm). Nevertheless, the differences among the results of all the methods are quite small, except for the OKR which gives the worst vertical accuracy. Moreover, the deterministic methods still give comparatively good vertical positioning accuracy. In terms of precision, the geostatistical methods (OKR and LSC) were noisier than the deterministic ones in both direction components (Figures 7-9).

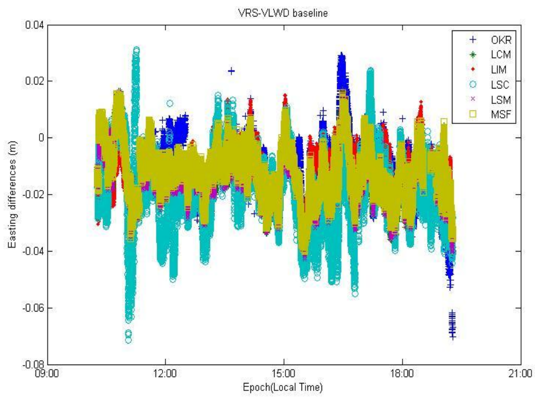

Figure 7: The offset from True Value of the User's Position in Local Topocentric Coordinates (Easting).

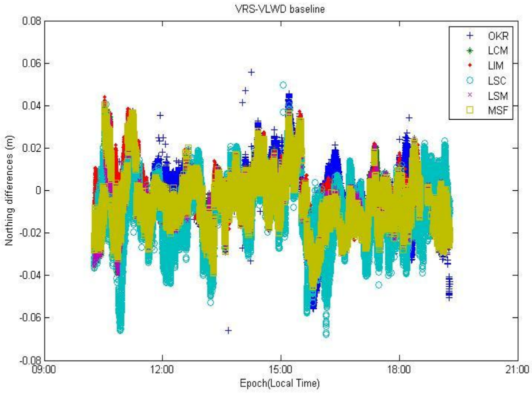

Figure 8: The offset from True Value of the User's Position in Local Topocentric (Northing).

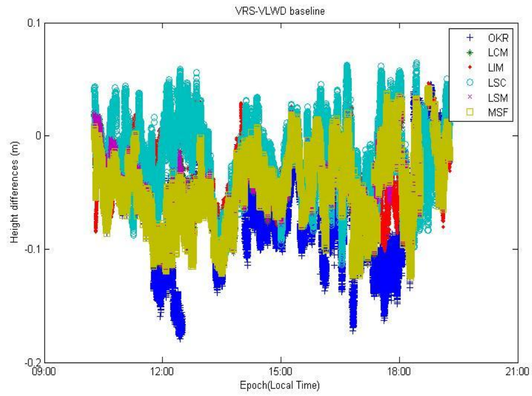

Figure 9: The offset from True Value of the User's Position in Local Topocentric (Height).

VRS-MGRV. The statistics of the VRS-MGRV test are computed and listed in Table 2. The offsets of the estimated positions from the true values are shown in Figures 10-12.

Table 2: Statistical results for the VRS user's positioning (MGRV) (horizontal and vertical components).

	RMSE (m)		Success rate (%)
	Hz	Vt	
OKR	0.022	0.309	100
LCM	0.022	0.352	100
LIM	0.022	0.348	100
LSC	0.041	0.294	75.9
LSM	0.022	0.352	100
MSF	0.055	0.339	97.1

These results indicate a slightly lower accuracy compared to the previous test. MSF amongst the deterministic methods and LSC amongst the geostatistical method performed badly in terms of horizontal accuracy (5.5cm and 4.1cm, respectively) whereas all other methods gave same better result (2.2cm). In the vertical component, the LSC is the best (29.4cm), followed by OKR with 30.9cm whereas the values from the other methods lay in the range of about 34-35cm.

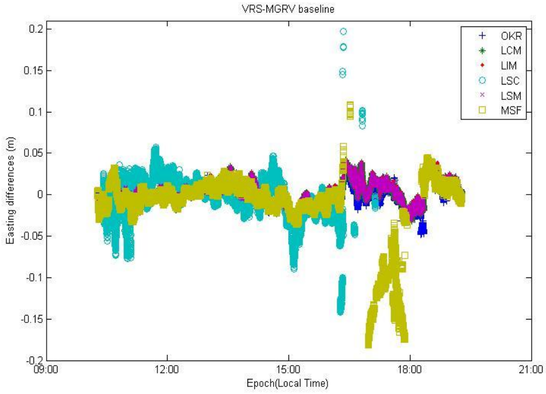

Figure 10: The offset from True Value of the User's Position in Local Topocentric Coordinates (Easting).

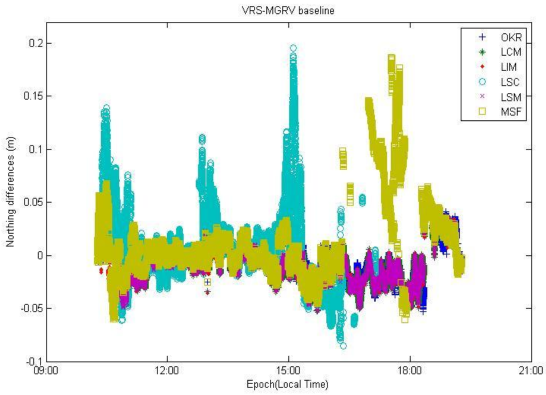

Figure 11: The offset from True Value of the User's Position in Local Topocentric Coordinates (Northing).

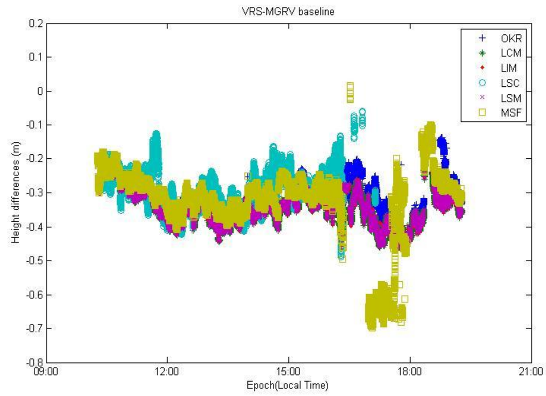

Figure 12: The offset from True Value of the User's Position in Local Topocentric Coordinates (Height).

Similar to the accuracy results, Figures 10-12 show the offsets from MSF and LSC which exhibit some outliers up to 20cm of errors in the horizontal and the vertical directions. All interpolation methods achieved 100% of success rate except those two methods with LSC (geostatistical method) being the lowest successful method to fix ambiguities (about 75.9%). Some outliers are noted from MSF and LSC, which may be caused by ambiguities being fixed not to the right values. The vertical component results suggest that the interpolation methods could not model the atmospheric errors accurately. This may be due to the extrapolation effect.

VRS-SPWD. The procedures the same as used in the previous two tests were used and the test results are shown in Table 3 and in Figures 13-15.

Table 3: Statistical results for the VRS user's positioning (SPWD) (horizontal and vertical components).

	RMSE (m)		Success rate (%)
	Hz	Vt	
OKR	0.045	0.306	73.7
LCM	0.065	0.339	49.1
LIM	0.037	0.233	73.0
LSC	0.047	0.320	34.2
LSM	0.065	0.339	49.2
MSF	0.037	0.313	99.9

The highest (3.7cm) and the lowest (6.5cm) horizontal accuracies were achieved by the deterministic methods. LIM and MSF produced the highest and the lowest were by LCM and LSM. OKR and LSC achieved 4.5cm and 4.7cm of horizontal accuracy, respectively. In the vertical direction, LIM was the best with 23.3cm, followed by OKR with 30.6cm. The LCM and LSM produced the same accuracy. The highest success rate was achieved by MSF (99.9%), followed by OKR and LIM with 73.7 and 73%, respectively. The success rate by the rest of the other methods was below 50%.

The offsets in local topocentric coordinates from all methods are larger compared to the previous tests. This is due to the fact that this network geometry is untypical case in which the rover is located a long away outside the network coverage and the rover is more distant from the master station.

The horizontal accuracy from both geostatistical and deterministic methods decreased when extrapolation process was applied compared to interpolation case even though this is not clear in the case of the second network geometry. The same holds for the effect of the baseline length between the rover and the master station. In the vertical direction, the rover location has clear effect on the accuracy of both method types.

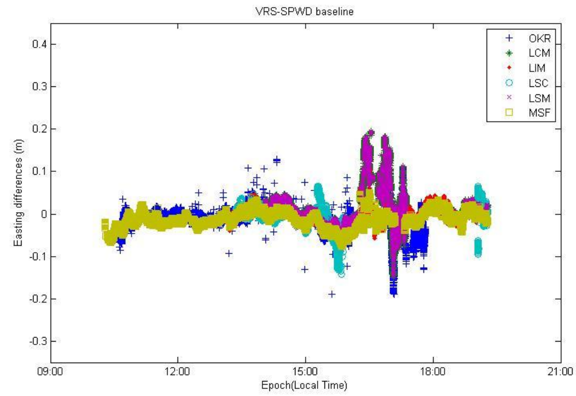

Figure 13: The offset from True Value of the User's Position in Local Topocentric Coordinates (Easting).

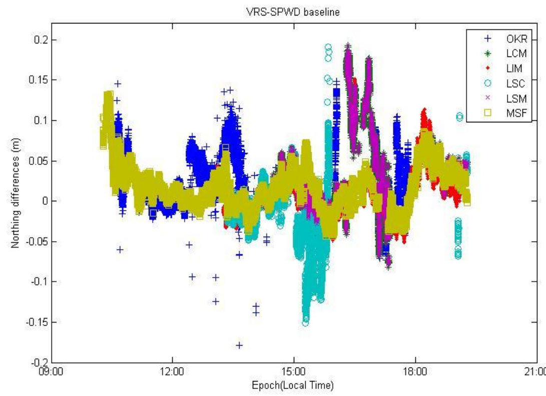

Figure 14: The offset from True Value of the User's Position in Local Topocentric Coordinates (Northing).

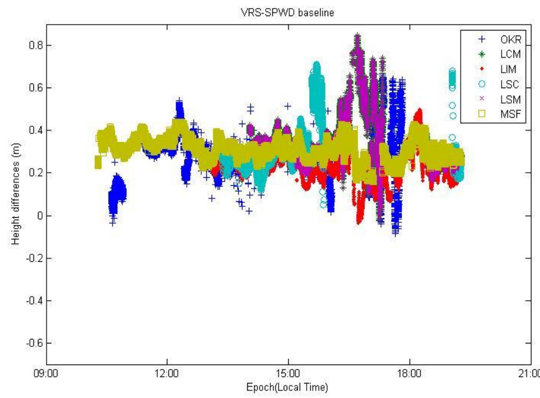

Figure 15: The offset from True Value of the User's Position in Local Topocentric Coordinates (Height).

Among all methods (Figures 16-17), the deterministic method LIM produced slightly better horizontal accuracy over all network geometries whereas better vertical accuracy was achieved by the geostatistical method LSC except for the third network geometry where LIM made the highest. In general, geostatistical methods gave comparable results to the deterministic methods with maximum difference of 2cm and 5cm in the horizontal and vertical direction, respectively. However the LIM generally outperformed all methods over all tests except for netGeometry 2 in the vertical direction with about 5cm difference to LSC.

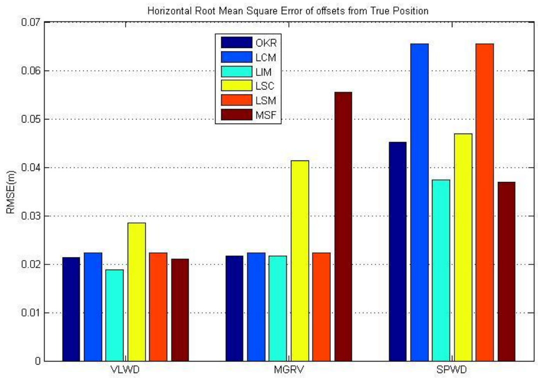

Figure 16: Root Mean Square Errors of horizontal positions of all methods and baselines.

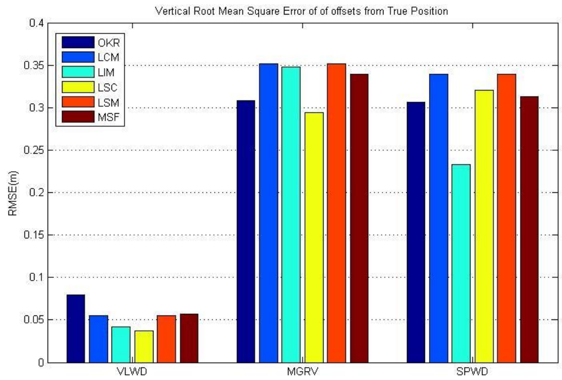

Figure 17: Root Mean Square Errors of vertical positions of all methods and baselines.

5. Concluding Remarks

A comprehensive study of different interpolation methods for calculating corrections for user's location with a multi-reference station GPS network applying the virtual reference station (VRS) approach was carried out. Six interpolation techniques (OKR, LSC, LCM, LIM, LSM and MSF), which are divided into two classes i.e., geostatistical and deterministic interpolation methods, were compared using three different network geometries for different testing cases.

In general, comparable results were obtained from both deterministic and geostatistical interpolation methods. Only the LIM shows slightly better performance in the horizontal directions for all tests whereas LSC in the vertical component in the first two cases. This investigation can hardly offer a confident suggestion about which interpolation method is the best. However, this paper assessed the performance of the interpolation methods under different network geometries in the geographic region of the study. From the results, the deterministic method (LIM) could be a better choice among both the deterministic and geostatistical methods. In the geostatistical methods, no big difference between OKR and LSC has been noted, however; a user may prefer LSC because of its simple implementation. Between the deterministic and geostatistical methods, the deterministic types are sufficient to model atmospheric biases for relatively small networks (<10 stations). Further tests should be carried out to investigate the accuracy decrease in the vertical component.

Acknowledgments

The first author is grateful to the scholarship provider, the Saudi Higher Education Ministry, and especially the University of Umm Al-Qura, Saudi Arabia.

References

Al-Shaery, A., S. Lim & C. Rizos (2010a) Functional models of ordinary kiging for real-time kinematic positioning. *5th International Navigation Conference and Exhibition-Melaha 2010*, Cairo, Egypt, 3-5 May, paper 2, CD-ROM procs.

Al-Shaery, A., S. Lim & C. Rizos (2010b) Functional models of ordinary kriging for medium range real-time kinematic positioning based on the virtual reference station technique. *23rd International Technical Meeting of The Satellite Division of the Institute of Navigation*, Portland, Oregon, 21-24 September, 2513-2521.

Alves, P.R.S. (2004) *Development of two carrier phase-based methods for multiple reference station positions.* Ph.D. Thesis, The University of Calgary.

Borga, M. & A. Vizzaccaro (1997) On the interpolation of hydrologic variables: formal equivalence of multiquadratic surface fitting and kriging. *Journal of Hydrology*, 195(1-4), 160-171.

Cannon, M.E. & G. Fotopoulos (2001) An overview of multi-reference station methods for cm-level positioning. *GPS Solutions*, 4(3), 1 -10.

Chen, H.Y., S. Han & C. Rizos (2000) Improving real-time positioning efficiency using the Singapore integrated multiple reference station network(SIMRSN). *13th International Technical*

Meeting of the Satellite Division of the U.S. Institution of Navigation, Salt Lake City, Utah, 19-22 September, 9-16.

Collier, P.A. (1988) *Integrated Geodesy Applied to Precise Engineering Surveys.* Ph.D. Thesis, The University of Melbourne.

Colombo, O.L., M. Hernandez-pajares, J.M. Juan, J. Sanz & J. Talaya (1999) Resolving carrier-phase ambiguities on-the-fly, at more than 100Km from nearest reference site, with the help of ionospheric tomography. *12th International Technical Meeting of the Satellite Division of the U.S. Institution of Navigation*, Nashville, Tennessee, 14-17 September, 1635-1642.

Cressie, N. (1993) Statistics for Spatial Data, New York: Wiley.

Dai, L., J. Wang, C. Rizos & S. Han (2001) Real-time carrier phase ambiguity resolution for GPS/GLONASS reference station networks. *International Symposium on Kinematic Systems in Geodesy, Geomatics & Navigation (KIS2001)*, Banff, Canada, 5-8 June, 475-481.

Dai, L., S. Han, J. Wang & C. Rizos (2003) Comparison of interpolation algorithms in network-based GPS techniques. *Journal of Navigation*, 50(4), 277-293.

Fortez, L. (2002) *Optimising the use of GPS multi-referenec stations for kinematic positioning.* Ph.D. Thesis, The university of Calgary.

Fotopoulos, G. & M.E. Cannon (2000) Spatial and temporal characteristics of DGPS carrier phase errors over a regional network. *56th Annual Meeting of the U.S. Institute of Navigation*, San Diego, California, 26-28 June, 54-64.

Gao, Y., Z. Li & J.F. McLellan (1997) Carrier phase based regional area differential GPS for decimeter-level positioning and navigation. *10th International Technical Meeting of the Satellite Division of the U.S. Institute of Navigation*, Kansas City, Missouri, 16-19 September, 1305-1313.

Gao, Y. & Z.Z. Liu (2002) Precise ionosphere modeling using regional GPS network data. *Journal of Global Positioning Systems*, 1(1), 18-24.

Geisler, I. (2006) Performance improvement of network RTK positioning. *National Technical Meeting of the Institute of Navigation*, Monterey, California, 18-20 January, 869-880.

Grgich, P.M., A. Kealy, C. Gordini, M. Hale & G. Retscher (2006) Atmospheric bias interpolation for network RTK GPS. *International Global Navigation Satellite Systems Society Symposium-IGNSS,* Gold Coast, Australia, 17-12 July.

Han, S. & C. Rizos (1996) GPS network design and error mitigation for real-time continuous array monitoring systems. *9th International Technical Meeting of the Satellite Division of the U.S. Institution of Navigation*, Kansas City, Missouri, 17-20 September, 1827-1836.

Han, S. & C. Rizos (1998) Instantaneous ambiguity resolution for medium-range GPS kinematic positioning using multiple reference stations. *Advances in Positioning and Reference Frames*, Rio de Janeiro, Brazil, 3-9 September, 283-288.

Hardy, R.L. (1971) Multiquadratic equations of topography and other irregular surfaces. *Journal of Geophysical Research*, 76(8), 1905-1915.

Hardy, R.L. (1984) Kriging, collocation, and biharmonic models for applications in the earth sciences (what's the difference?). *44th Annual Meeting of the American Congress on Surveying and Mapping.*, Washington DC, USA, 11-16 March, 363-372.

Hofmann-Wellenhof, B., H. Litchtenegger & E. Wasle (2008) GNSS - Global Navigation Satellite Systems: GPS, GLONASS, Galileo and More, Vienna: Springer-Verlag.

Kitanidis, P.K. (1997) Introduction to geostatistics : applications to hydrogeology, Cambridge: Cambridge University Press.

Krarup, T. (1970) The method of least squares collocation. *Studia Geophysica et Geodaetica*, 14(2), 107-109.

Lee, P.S., P.P. Lynn & E.M. Shaw (1974) Comparison of multiquadric surfaces for the estimation of area rainfall. *Bulletin of the International Association of Scientific Hydrology*, 19(3), 303 - 317.

Moore, R.J., B.C. Watson, D.A. Jones, K.B. Black, C.M. Haggett, M.A. Crees & C. Richards (1989) Towards an improved system for weather radar calibration and rainfall forecasting using raingauge data from a regional telemetry system. *In*: Kavvas, L.M. (ed.) New Directions for Surface Water Modelling, IAHS 181, 13-21.

Raquet, J.F. (1997) Multiple user network carrier-phase smbiguity resolution *International Symposium on Kinematic Systems in Geodesy, Geomatics & Navigation (KIS1997)*, Banff, Canada, 3-6 June, 45-55.

Raquet, J.F. (1998) *Development of a method for kinematic GPS carrier-phase smbiguity resolution using multiple reference receivers.* Ph.D. Thesis, The University of Calgary.

Rizos, C. (2002) Network RTK research and implementation - A geodetic perspective. *Journal of Global Positioning Systems*, 1(2), 144-150.

Shaw, E.M. (1994) Hydrology in Practice, GBR: Spon Press.

Van der Marel, H. (1998) Virtual GPS reference stations in the Netherlands. *11th International Technical Meeting of the Satellite Division of the U.S. Institution of Navigation*, Nashville, Tennessee, 15-18 September, 49-58.

Vollath, U., A. Buecherl, H. Landau, C. Pagels & B. Wanger (2000) Multi-base RTK positioning using Virtual Reference Stations. *13th International Technical Meeting of the Satellite Division of the U.S. Institution of Navigation*, Salt Lake City, Utah, 19-22 September, 1061-1070.

Wackernagel, H. (2003) Multivariate Geostatistics: An Introduction with Application, Berlin: Springer.

Wanninger, L. (1995) Improved ambiguity resolution by regional differential modelling of the ionosphere *8th International Technical Meeting of the Satellite Division of the U.S. Institute of Navigation*, Palm Springs, California, 1-15 September, 55-62.

Wielgosz, P., D. Grejner-Brzezinska & I. Kashani (2003) Regional ionophere mapping with kriging and multiquadric methods. *Journal of Global Positioning Systems*, 2(1), 48-55.

Wu, S. (2009) *Performance of regional atmospheric error models for NRTK in GPSnet and the implementation of a NRTK system.* PhD, RMIT University.

Wubbena, G., A. Bagge, G. Seeber, V. Boder & P. Hankemeier (1996) Reducing distance dependent errors for real-time precise DGPS applications by establishing reference station networks. *9th International Technical Meeting of the Satellite Division of the U.S. Institute of Navigation*, Kensas City, Missouri, 17-20 September, 1845-1852.

Yi, Y. & D. Grejner-Brzezinska (2003) Kinematic carrier phase gps positioning aided by an instantaneous local ionospheric model based on multiple base stations using kriging. *59th Annual Meeting of the U.S. Institute of Navigation* Albuquerque, New Mexico, 23 - 25 June, 387 - 396.

Zhang, S., S. Lim, C. Rizos & J. Guo (2009) Atmospheric decomposition for VRS based network-RTK system. *22nd International Technical Meeting of the Satellite Division of the U.S. Institution of Navigation*, Savannah, Georgia, 22-25 September, 2707-2716.

Biographies

Ali Al-Shaery is currently a doctoral student in the School of Surveying and Spatial Information Systems at the University of New South Wales, Australia and assistant lecturer at the University of Umm Al-Qura, Saudi Arabia. He obtained his B.Sc. degree in Civil Engineering from the University of Umm Al-Qura, Saudi Arabia and M.Sc. degree from University College London, United Kingdom in 2003 and 2007, respectively. His current research interests are mainly GNSS network-RTK algorithms and applications.

Samsung Lim is an Associate Professor in the School of Surveying and Spatial Information Systems at the University of New South Wales, Australia. Samsung is a graduate of Seoul National University, South Korea, with a Bachelor's degree from the Department of Mathematics in 1988, and a Master's degree from the same department in 1990. Samsung obtained his PhD degree from the Department of Aerospace Engineering & Engineering Mechanics in the University of Texas at Austin in 1995. Samsung's career total publication includes over 130 peer-reviewed papers, over 60 presentations at national and international conferences, and over 40 invited seminars at various research organisations and universities in the world. Samsung is a senior member of IEEE.

Chris Rizos is currently Professor and Head of School. Chris has been researching the technology and high precision applications of GPS since 1985, and has published over 500 journal and conference papers. He is a Fellow of the Australian Institute of Navigation, a Fellow of the U.S. Institute of Navigation, and a Fellow of the International Association of Geodesy (IAG). He is currently the President of the IAG and a member of the Governing Board of the International GNSS Service.