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Statistical Comparison of Various Interpolation Algorithms for Grid-Based Single Shell Ionospheric Model over Indian Region

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Abstract

Future Satellite Based Navigation Systems based on GPS, like US Wide Area Augmentation Sysytem (WAAS), are of global interest to the scientific community. Precise estimation of ionospheric delay is most crucial for successful implementation of the systems. Due to the complex ionospheric structure and large variation of Total Electron Content (TEC) in low latitudes, it is necessary to compare and validate the efficiency of the existing algorithms in this region. In this study, performances of the various interpolation algorithms for TEC calculation at ionospheric grid point (IGP) and user position for grid-based Single Shell Model have been tested for 72 test days of 2005. Based on the results obtained from this analysis, it has been found that, for the Indian region, it would be more suitable to use Ordinary Kriging in place of Planar Fit to estimate delay at an IGP (as used by US WAAS). It has also been found that Ordinary Kriging performs better than the Bilinear Interpolation technique at the user end.

Keywords: GAGAN, Ionosphere, Interpolation algorithm, Single Shell Model

1. Introduction

Satellite Based Augmentation Systems (SBAS) for airline navigation are currently under development worldwide. United States was the first country to develop and demonstrate the Wide Area Augmentation System (WAAS, 2008, Walter T, 2002 and http://waas.stanford.edu.). Later on, Europe started European Geostationary Navigation Overlay Service (EGNOS) program and similar programs are at the various stages of development in the countries like Japan, China, Brazil and India. Indian Space Research Organization (ISRO) is involved in the development and establishment of a full complement of Wide Area Augmentation System (WAAS) over Indian Airspace, called GAGAN (GPS Aided Geo Augmented Navigation), in collaboration with Airports Authority of India (AAI).

The primary goal of SBAS is to enhance the accuracy and integrity of user position estimates based upon Global Positioning System (GPS) measurements. In the absence of the selective availability, the ionosphere is the largest source of errors for single-frequency users of GPS. Currently, the ionospheric models used in SBAS rely on the Single-Thin- Shell approximation (Klobuchar,1987; Mannucci et al.,1998). WAAS derives slant ionospheric delay error and confidence bounds from estimates of vertical delay modelled on a grid at regularly spaced intervals of latitude and longitude (5x5 degree). The vertical delay estimates at each IGP is calculated from a Planar Fit technique using measured delay values surrounding that IGP. User estimates delay at its position using Bilinear Interpolation from the delay values at four corners of the grid in which it lies.

Due to presence of complex ionospheric structure and the Equatorial Ionospheric Anomaly over the Indian region (Rama Rao et al, 2006), it is required to test the performance of grid-based thin-shell model with different user Interpolation techniques. In order to find a grid-based model suitable for the Indian region, performance study of the algorithms over large time period is essential. Present analysis has been done for all quiet days (A_P index <50) of the year 2005, and an attempt has been made to find out an algorithm which performs well for the Indian region under these conditions. Choice of testing under quiet conditions was made only to exclude the effects due to various abrupt variations derived from enhanced geomagnetic activities. Issues related to the enhanced geomagnetic activities are beyond the scope of this paper.

In the present study, grid-based single-shell model using Planar Fit (as used by WAAS), Ordinary Kriging and Universal Kriging have been attempted to obtain the vertical ionospheric delay values at the IGPs over the Indian region. In this study, besides Bilinear Interpolation Kriging (Ordinary and Universal) interpolation and Planar Fit has also been used to determine user ionospheric delays. A statistical comparison and validation of these different algorithms has been done by obtaining the Root Mean Square Error (RMSE) between observed (truth data) and calculated slant ionospheric delay values.

In the following sections, an overview of the grid based estimation of the ionospheric delay by Single Shell Model with a brief description of the different techniques used at an IGP and Ionospheric Pierce Point (IPP) is given.

2. Grid Based Single Shell Model:

GPS signal delay caused by the ionosphere is directly proportional to the number of the free electrons along the signal ray path. The total electron content (TEC) along the ray path of the signal from satellite to receiver may be written as the path integral of the electron density along with the line of sight.

$$TEC = \int_S^R Ne(l)dl \quad (1)$$

where the subscripts S and R identify, respectively, the satellite and receiver in question, and Ne is the electron density.

Thus, solving for slant delay is an inherently three dimensional problem. The Single-Shell Model reduces this three dimensional problem to two-dimensions by introducing the simplifying assumption that the whole ionosphere is compressed only in a neighbourhood of a fixed altitude, taken as 350 km (RTCA special committee 159, 2001, Gao Y. and Liu Z.,2002).

The relation given below allows to estimate the vertical delay at the IPP to be inferred from a measurement of slant delay. Mathematically, it is expressed as:

$$TEC_{slant} = M(E,h).TEC_{Vertical} \quad (2)$$

where

$$M(E,h) = [1 - (\mathrm{Re}.\cos E/(\mathrm{Re}+h))^2]^{-1/2} \quad (3)$$

is a mapping function. Where E , h and R_e denote elevation angle, maximum electron density altitude and radius of the Earth respectively. After determining the vertical delays at IGP, user interpolates the vertical delay at its position using these delay values. Using mapping function, these vertical delays are converted to slant delays (RTCA special committee 159, 2001; Mishra and Enge, 2001).

2.1. Planar Fit Model

With Planar Fit method, a smoothened surface of delay variations is obtained by averaging highly variable spatial data (IPP delays). Optimal weights for IPPs are calculated based on the residual errors. The residual error is obtained by using the correlation structure, which includes measurement variances of IPP, de-correlation factor and biases forming weight matrix (Walter et al, 2000).

The ionospheric delay around an IGP is estimated using Planar Fit linear function given as:

$$I_{y,IGP}(x,y) = a_0 + a_1 x + a_2 y \quad (4)$$

Here, a_0 represents exact vertical delay at the IGP, a_1 and a_2 represents delay variation along X and Y axes respectively.

The solution for Planar coefficients (a_0 , a_1 , a_2) are obtained by the weighted least square method and is given as,

$$[a_0 \quad a_1 \quad a_2] = [(G^T.W.G)^{-1}.G^T.W.I_{y,IPP}] \quad (5)$$

where $I_{y,IPP}$ is the Matrix of measured vertical delay at IPPs surrounding an IGP, G is the observation matrix and W is the weight matrix.

As for the calculation of W , the correlation coefficient value for the Indian region is taken as four (Acharya etal,2006). Since IGP is located at the origin in the local coordinate system, vertical delay at IGP can be obtained by determining Planar coefficient a_0 , making other coefficients zero. Finally, the delay estimate at the IGP is given by (Walter et al, 2000)

$$I_{y,IPP} = [1 \quad 0 \quad 0].[(G^T.W.G)^{-1}.G^T.W.I_{y,IPP}] \quad (6)$$

2.2. Ordinary Kriging

It has been observed that the nominal ionosphere can be well characterized by a Planar trend with a covariance depending on the distance, but because of irregular behaviour of the ionosphere, it is inappropriate to estimate delay values using Planar Fit. Therefore another technique called Kriging, has been implemented (Wielgosz P. et al, 2003). This uses a slack variable, called Lagrange multiplier λ to obtain the delay in an optimum and unbiased manner. Kriging is a geo-statistical interpolation technique that considers both the distance and the degree of variation between known data points when estimating values in unknown areas. A kriged estimate is a weighted linear combination of the known sample values around the point to be estimated (Davis J. C., 1986). Applied properly, Kriging allows the user to derive weights that result in optimal and unbiased estimates. In this technique, the ionospheric estimation is assumed to be a statistical process and the relation between the statistical variables is defined by:

$$\mathrm{var}(Z(x+h) - Z(x)) = 2\gamma h \tag{7}$$

where, $Z(x+h)$ and $Z(x)$ are two spatially separated random outcomes, γ is called the semivariogram. The variogram of measurements is only a function of the distance between measurements. Thus, before the actual interpolation can begin, Kriging must calculate every possible distance weighing function. This is done by generating the experimental semivariogram of the data set and choosing a mathematical model which best approximates the shape of the semivariogram. Spherical model of the semivariogram has been used here and its mathematical form is given as follows:

$$\gamma(h) = c\left(\left(\frac{3d}{2a}\right) - \left(\frac{d}{2a}\right)^3\right), \text{ when } d \le a \tag{8a}$$

$$= c \qquad , \text{ when } d > a \tag{8b}$$

where, d is any pre-defined distance, h is the distance between estimated location and observed location. The semivariances have been calculated for different values of h and have been plotted in the form of a semivariogram. If the difference of different values of h is increased the points being compared are less and less closely related to each other and their difference becomes larger, resulting in larger values of $\gamma(h)$. At some distance, the points being compared are so far apart that they are not related to each other. And their squared differences become equal in magnitude to the variance around the average value. The semivariance no longer increases and the semivariogram developed a flat region, called a sill c . The distance at which the semivariance approaches the variance is referred to as range (a) of the rationalised variable (Davis J. C., 1986).

Finally, the delay estimate at IGP, $I_{est}(x)$, is given by

$$I_{est}(x) = \sum_{k=1}^{n} W_k . I_{mes}(x_k) \tag{9}$$

where W_k is the weight factor depending on the distance and $I_{mes}(x_k)$ is the measured ionospheric delay at the k[th] IPP (Cressie, 1993).

2.3. Universal Kriging

In the presence of a trend or slow change in the average value, as in case of low latitude ionosphere, a linear estimator is no longer unbiased. Universal Kriging can then be used and can be regarded as consisting of three operations: first the drift must be estimated and removed. Then the stationary residuals are Kriged to obtain the needed estimates. Finally, the estimated residuals are combined with the drift to obtain the estimates of the actual surface. Therefore, Kriging with a trend or Universal Kriging is the technique, which minimizes the mean-square estimation error given that the spatial structure is given by (Blanch J, 2003)

$$I_y(x,y) = a_0 + a_1 x + a_2 y + R(x,y) \tag{10}$$

where the first three terms define the Planar trend and $R(x,y)$ is a random function with zero mean and a given variogram. The expected value of delay estimate is given by:

$$E(I_{est}(x)) = E\left(\sum_{k=1}^{n} \lambda_k . I_{mes}(x_k)\right) \tag{11}$$

Now, the following equation can be simplified

$$E(a_0 + a_1 x^{est} + a_2 x^{north}) = E(\lambda_1 . \mathrm{Im}\, ea(x_1) + \lambda_2 . \mathrm{Im}\, ea(x_2) + ... + \lambda_n . \mathrm{Im}\, ea(x_n)) \tag{12}$$

To get the following form

$$X = \lambda . G^T \tag{13}$$

where G is the geometry matrix, X is the position geometry of the IGP along the east and the north directions and is given as:

$$X = [1 \quad x^{est} \quad x^{north}] \tag{14}$$

The final aim is to find λ such that it minimises the difference between the estimated and the true values at the IGP. λ is obtained by applying the optimization using Lagrange Multiplier and is given as:

$$\lambda = (W - W.G(G^T.W.G)^{-1}.G^T.W)C(x,x_k) + W.G(G^T.W.G)^{-1}.X \tag{15}$$

where, $W = (C(x_i, x_j))^{-1}$

$C(x_i, x_j)$ is an nxn matrix whose elements are computed using the assumed covariance for the process noise. $C(x, x_k)$ is a vector whose elements are the covariance between the residual at the IGP location and each of the residuals at the measurement location.

Finally, vertical delay at the IGP is obtained by (Blanch, 2003),

$$I_{est}(x) = \sum_{k=1}^{n} \lambda_k . I_{mes}(x_k) \tag{16}$$

Here, an exponential model of the semivariogram has been used to calculate c (Blanch J, 2003).

2.4. Bilinear Interpolation

For four-point interpolation, the mathematical formulation for interpolated vertical IPP delay as a function of IPP latitude and longitude is provided in RTCA document using Bilinear Interpolation (RTCA special committee 159, 2001).

3. Data Collection, Pre-processing and Analysis

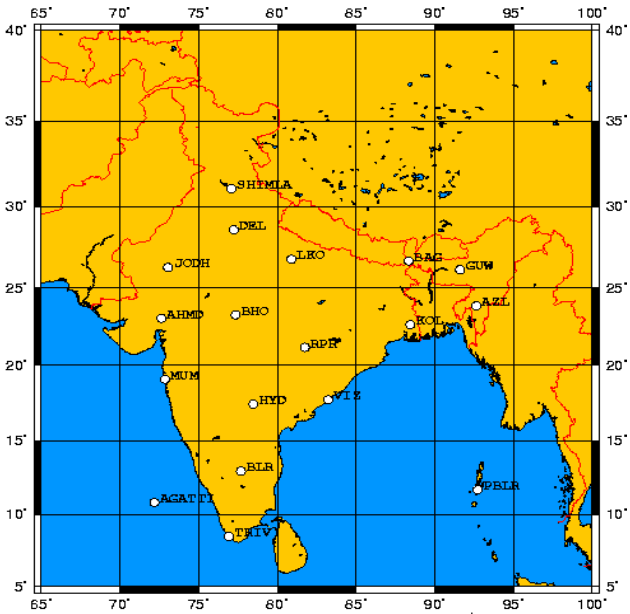

Fig. 1 Map of the TEC stations collecting data for GAGAN

To carry out the comparison and validation of different ionospheric models for GAGAN, TEC data has been collected from 18 TEC stations having dual frequency receivers, spread all over the Indian region (5ºN-35ºN degrees latitude and 70ºE -95ºE degrees longitude). Locations of the TEC stations are shown in Figure 1. These stations are selected nearly at the centres of the eighteen 5ºx5º grids over India. GPS data has been collected with an elevation angle greater than 15 degrees from each of the 18 stations and has been used to calculate TEC values for the present study. These receivers receive pseudorange and carrier phase data from all the visible satellites above the said cut off elevation angle. TEC has been calculated from the data collected over the one minute interval and recorded on a PC.

Before using the TEC data, receiver bias has been removed by using Kalman Filter. Satellite bias correction has also been applied. A brief description of pre-processing and analysis is given as follows:

3.1. TEC estimation

Raw pseudo range and carrier phase data is recorded in .GPS file at 5 Hz interval in dual frequency GPS receiver (Novatel OEM4). Then the .GPS file is converted into RINEX format using a software to get Observation and Navigation files. Thus, we get C_1 and P_2 (pseudo range in meters) and $f1$ and $f2$ (carrier phase in cycles) from L1 and L2 of all visible satellites from the Observation file.

TEC can be estimated from relative code delay and also from relative carrier phase measurements at L_1 and L_2 using dual frequency GPS receiver.
In our analysis estimation of TEC has been done using pseudorange measurement and the relation can be summarized as follows,

$$TEC_{\rho} = 9.5196(P_2 - C_1) \tag{17}$$

where C_1 and P_2 are the pseudoranges in meters using C/A code measurement and P code measurement respectively.

Similar expression with carrier phase measurements for estimating TEC may be given as:

$$TEC_{\Phi} = \frac{(\Phi_2\lambda_2 - \Phi_1\lambda_1) + (N_{f2}\lambda_2 - N_{f1}\lambda_1)}{1.34.10^{-7}(\lambda_2 / f_2 - \lambda_1 / f_1)}$$
$$TEC_{\Phi} = 9.5499\{ (\Phi_2\lambda_2 - \Phi_1\lambda_1) + (N_{f2}\lambda_2 - N_{f1}\lambda_1) \} \tag{18}$$

Here, $(\Phi_2\lambda_2 - \Phi_1\lambda_1)$ is in meters, λ_1 and λ_2 are wavelengths at $f1$ and $f2$, and N_{f1} and N_{f2} are integer ambiguity terms at $f1$ and $f2$ respectively.

The above expression (18) is approximately equal to the expression obtained for TEC from the pseudorange (code) difference measurements given in equation (17) except for the integer ambiguity term.

It is well known fact that carrier phase can be measured more precisely than code measurement and so the TEC (Misra P. and Enge P. , 2001). But, carrier phase contains integer ambiguity and code based measurements are noisy. So, code smoothening has been done using the carrier phase measurements, while determining TEC, and this does not involve solving for the integer ambiguity in carrier phase measurements.

To get noise free and absolute TEC, carrier smoothing is done by moving average of the difference between TEC from code and carrier phase calculated at each epoch and added to the carrier phase TEC and is expressed as:

$$TEC = TEC_{\Phi} + (1/K)\sum_{k=0}^{K} (TEC_{\Phi}^{k} - TEC_{\rho}^{k}) \quad (19)$$

where k is the epoch time. The smoothed carrier phase TEC fitted to the level of code TEC to get smoothed code TEC.

Satelite and receiver Inter Frequency Biases (IFB) have also been estimated and removed while calculating TEC. Satellite IFB includes differential P_1 - P_2 code bias and the differential P_1 - C_1 code bias, which are estimated and measured separately. Differential P_1 - P_2 code bias can be determined from the GPS broadcast differential group delay (τ_{GD}) values by calculating GPS offset time and mean τ_{GD} values. Method of estimating P_1 - P_2 bias has been studied and working algorithm is developed. The P_1 - C_1 bias values are available on CODE website (//www.aiub.unibe.ch/ionosphere.html). These values are used and after converting into TEC units, it is also applied to the above equation. Receiver IFB is caused by the difference between the frequency response of filtering the GPS L_1 and L_2 signals, resulting in a time-misalignment of the two pseudoranges. Receiver IFB is not constant and varies with time (Sardon E. and Zarraoa N, 1997). In our approach, these biases have been estimated by Kalman Filter technique (Sardon et al, 1994). For more details on receiver bias correction one may refer to Rajat et al (2007).

Combining all the biases one can write the true TEC by (Wilson B.D. and Manuncci A. J, 1993, Sardon E. and Zarraoa N., 1997),

$$TEC_{true} = TEC + K_r + K^s \quad (20)$$

where, $K_r = R_{bias}$ is the receiver inter frequency bias and $K^s = P_1P_{2bias} - P_1C_{1bias}$ is the satellite bias. It can be expressed in more general form as follows,

$$TEC_{true} = TEC + P_1P_{2bias} - P_1C_{1bias} + R_{bias} \quad (21)$$

3.2. Analysis

For the present analysis, data from all the 18 TEC stations has been used. Analysis has been done for all quiet days of year 2005 with A_p index <50. A statistical validation of the different techniques, described below, has been done.

Bilinear Interpolation is implemented by WAAS to calculate the User Ionospheric Vertical Delay (UIVD) from the four grid values of grid ionosphere vertical delay (GIVD). But, in the present study, authors have also used Kriging (Ordinary and Universal) and Planar Fit techniques to estimate slant delay at user's position. In total, there are three different techniques implemented at the IGP which are Planar Fit, Ordinary Kriging and Universal Kriging and four different techniques at the IPP i.e. Planar Fit, Ordinary Kriging, Universal Kriging and Bilinear Interpolation. From these total six combinations: (a) Ordinary Kriging at IGP with Ordinary Kriging at IPP (b) Ordinary Kriging at IGP with Bilinear Interpolation at IPP (c) Universal Kriging at IGP with Universal Kriging at IPP (d) Universal Kriging at IGP with Bilinear Interpolation at IPP (e) Planar Fit at IGP with Planar Fit at IPP and (f) Planar Fit at IGP with Bilinear Interpolation at IPP, have been tested to find the combination which gives least RMSE in slant TEC (STEC) and highest percentage counts less than three TEC over the Indian region.

3.3. Validation Methodology

In the present study for validation purpose the methodology adopted is to remove each ray and reconstruct it separately with the help of remaining rays. Reconstruction is done in following two steps. First, ionospheric delay values at IGPs have been determined by using Single-Shell model with different algorithms without using that particular ray. In second step, using vertical delay values at the four corners of IGP in which the particular IPP lies, slant ionospheric delays (STEC) at IPP has been reconstructed using all the algorithms considered. In total six different combinations as described above has been analyzed. Yearly averaged Root Mean Square Error (RMSE) in STEC has been estimated for all quiet days of the year 2005. Following formula has been used for the calculation of the RMSE in Slant TEC values:

$$RMSE_{STEC} = \sqrt{\frac{\sum_{i=1}^{N} (STEC_m(i) - STEC_r(i))^2}{N}} \quad (22)$$

where $STEC_m$ is the measured (truth data) STEC, $STEC_r$ is the reconstructed STEC and N is the total number of IPPs.

4. Results and Discussion

The results are shown in the Figures 2 and 3. In Figure 2, middle curve represents the yearly average of the RMSE in STEC for different hours (00UT to 23 UT) with the maximum and minimum values shown by vertical lines. For having a better insight of the error variation, one-sigma value above and below the average has also been plotted along with the average curve. As one-sigma value around the average contains the 68 % of the total points, it can be concluded that the maximum errors occurs only for limited cases and average curve represents the true nature of the errors. All the combinations show significant variation of the errors from the mean line. This is shown by the error bars in Figure 2. Difference in maximum and minimum error values is particularly very large for the Universal Kriging and Planar Fit techniques in comparison with the Ordinary Kriging. This shows the consistency and reliability of the Ordinary Kriging over the other considered algorithms. These results are expected as the equatorial ionosphere varies vary rapidly.

Further, from Figure 2, it can be seen that most of the time maximum error in STEC occurs in the peak hours i.e., from 6-10 UT. This is the time period in which ionospheric anomaly is prominent over the Indian region. Also it is clearly visible that 'Ordinary Kriging at IGP with Ordinary Kriging at IPP' and 'Ordinary Kriging at IGP with Bilinear Interpolation at IPP' performs better than all other combinations considered. Further, Ordinary Kriging estimates the delay more reliably than Bilinear Interpolation technique at IPPs for all the time. This clearly indicates that the ionospheric delay over Indian region can well be modelled with Ordinary Kriging both at IGP and IPP in comparison to other techniques.

Although Universal Kriging technique is theoretically more sound than the Ordinary Kriging, it does not show the improvement over the Ordinary Kriging. Authors feel that the reason for this is the use of a constant correlation coefficient in Universal Kriging, which has been taken as four (Acharya et al, 2006), may not be appropriate for the Indian region and should have been taken as variable.

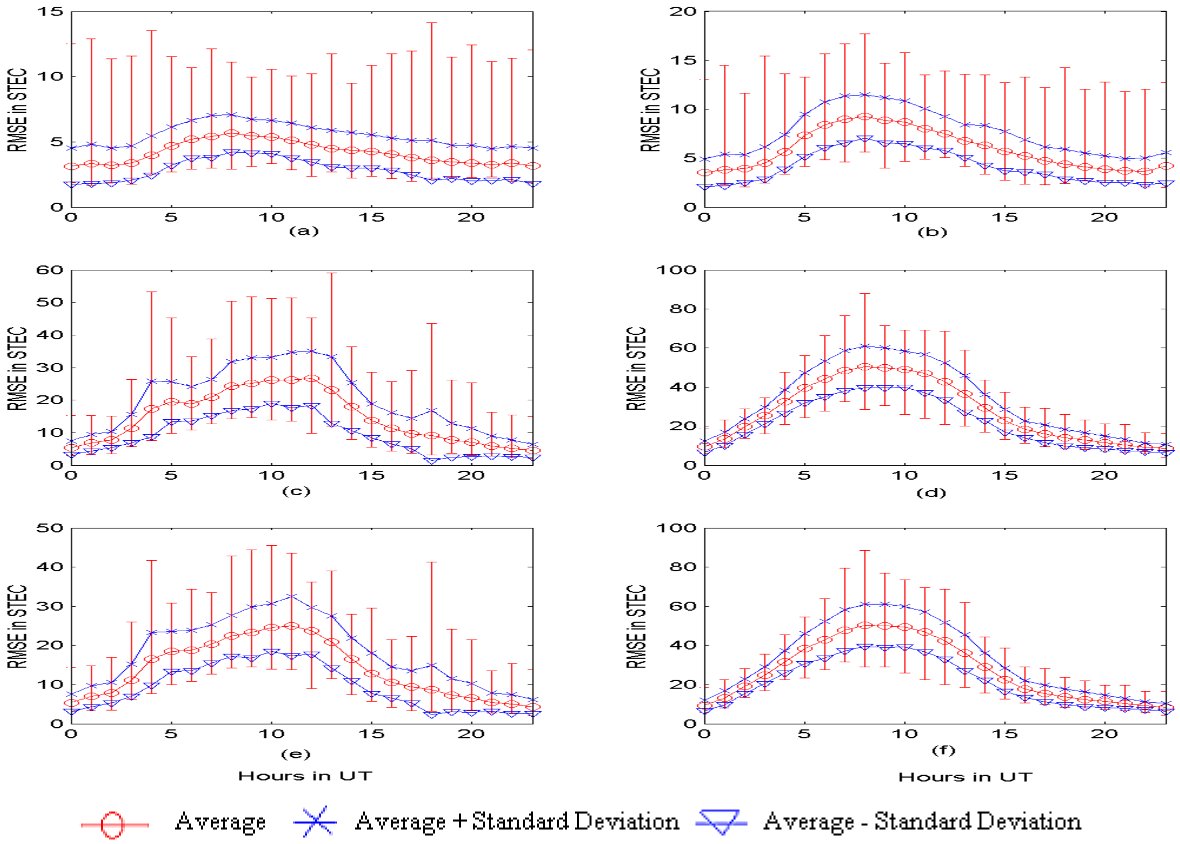

Fig. 2 Variation of hourly RMSE in STEC with (a) Ordinary Kriging at IGP and Ordinary Kriging at IPP, (b) Ordinary Kriging at IGP and Bilinear Interpolation at IPP, (c) Universal Kriging at IGP and Universal Kriging at IPP, (d) Universal Kriging at IGP and Bilinear Interpolation at IPP, (e) Planar Fit at IGP and Planar Fit at IPP, (f) Planar Fit at IGP and Bilinear Interpolation at IPP. All errors shown are in TEC units.

As an stringent accuracy criteria for the Indian region, it was required that the error should not exceed three TEC values (0.48 meters), therefore a comparison plot for percentage count between two best performing combinations have also been given in Figure 3.

'Ordinary Kriging at IGP with Ordinary Kriging at IPP' has shown highest percentage of counts for less than three TEC for all the 24 hours. For this combination, at 00 Hrs UT, percentage count less than three TEC is around 49, it decreases to one around 10 Hrs UT (around peak anomaly time) and again increases to 50 at 23 Hrs UT. During peak anomaly hours of the day percentage count (< 3 TEC) is very low.

From Figure 3, it can also be observed that none of the algorithms used in the single shell model satisfy the criteria required for GAGAN for all hours of the day. Grid spacing of 5x5 degree may be a possible reason for getting low percentage counts less than 3 TEC, which may improve by reducing the grid size for Indian region. Increasing the number of stations may also improve the percentage counts.

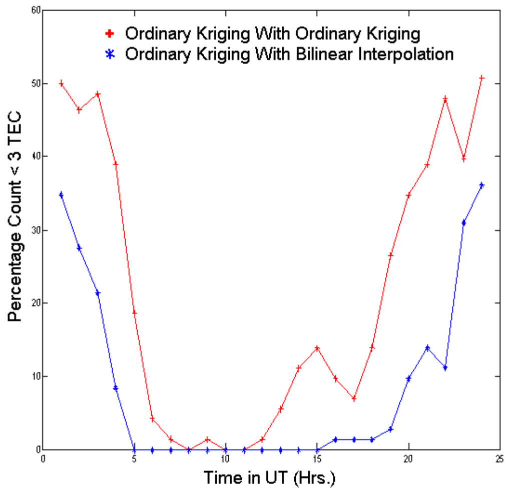

Fig. 3 Comparison of percentage count less than 3 TEC for Ordinary Kriging at IGP with Ordinary Kriging at IPP and Ordinary Kriging at IGP with Bilinear Interpolation at IPP at every hour (00-23 UT) for all the quiet days of 2005.

5. Conclusions

Preliminary study has been performed using different interpolation techniques for grid based Single Shell Model over Indian region for 72 quiet days of 2005 using GPS TEC measurement. It can be concluded from the present study that Planar Fit (as used by US WAAS) technique to calculate VTEC at IGPs may not be suitable for Indian region. On the basis of present study, it has been observed that Ordinary Kriging seems to be a better choice than Planar Fit technique for estimation of VTEC at IGP. Also, instead of Bilinear Interpolation technique at user end, Kriging Interpolation technique seems to be more appropriate for this region.

A more detailed study including disturbed days with more years data is undergoing to meet the GAGAN requirement.

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