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Three Carrier Ambiguity Resolutions: Generalised Problems, Models and Solutions

Yanming Feng[1] Bofeng Li[1, 2]
[1] *Faculty of Science and Technology, Queensland University of Technology, Brisbane Q4001, Australia*
[2] *Department of Surveying and Geo-informatics Engineering, Tongji University, Shanghai 200092, China*

Abstract

In this paper, the problems of three carrier phase ambiguity resolution (TCAR) and position estimation (PE) are generalised as real time GNSS data processing problems for a continuously observing network on large scale. In order to describe these problems, a general linear equation system is presented to uniform various geometry-free, geometry-based and geometry-constrained TCAR models, along with state transition questions between observation times. With this general formulation, generalised TCAR solutions are given to cover different real time GNSS data processing scenarios, and various simplified integer solutions, such as geometry-free rounding and geometry-based LAMBDA solutions with single and multiple -epoch measurements. In fact, various ambiguity resolution (AR) solutions differ in the floating ambiguity estimation and integer ambiguity search processes, but their theoretical equivalence remains under the same observational systems models and statistical assumptions. TCAR performance benefits as outlined from the data analyses in some recent literatures are reviewed, showing profound implications for the future GNSS development from both technology and application perspectives.

Keywords: Three carrier phase ambiguity resolution, real time GNSS data processing, Constrained Kalman filter.

1. Generalised AR Problems

Carrier phase ambiguity resolution (AR) consists of both float ambiguity estimation and integer ambiguity determination process in GNSS data processing. AR is one of the key enabling techniques for various precise GNSS applications using carrier phase measurements, although different AR models and methods are used in various real time and post-processing positioning problems. However, the dual-frequency based instant AR are basically restricted to real time kinematic (RTK) positioning services over a short baseline or local scale network, due to the effects of various distance-dependent biases. For a long baseline or regional to global network, AR or partial ambiguity resolution (PAR) is possible over a period of observation or longer data arcs, to provide precise data analysis solutions in near real time or post processing modes. An example for the real time regional and global differential positioning services is the OmniStar High-Performance (HP) services where position estimation (PE) is based on the floating ambiguity estimation of phase measurements; but the convergent time for decimetre accuracy takes up to tens of minutes.

In the context of future GNSS systems, three or multiple carrier ambiguity resolution (TCAR/MCAR) can potentially bring various existing GNSS services to a new level of performance at the local, regional and global scales (Feng & Rizos 2005; Hatch 2006). This is because with triple frequency code and phase signals accessible by civilian users, various frequency combinations will allow wider wide-lane combinations resulting in successful ambiguity resolution over much longer baselines. This result has many implications. First of all, in the network-RTK for centimetre positioning services, the inter-station distances would be extended from several tens to hundreds of kilometres. Feng and Li (2008a) demonstrated that use of triple frequency allows the inter-station distances to be roughly doubled in the network RTK services with respect to the dual-frequency based reference station spacing. Secondly, global real time decimetre positioning is possible. Comparing to the dual-frequency based differential positioning; a major benefit of using the third frequency signals is the reduction of convergence time at the 20 cm level RMS accuracy from a few tens of minutes to a few minutes. Thirdly, with ambiguity-fixed double differenced measurements over a globally distributed Continuously Operating Reference Stations (CORS), the scientific GNSS solutions, such as precise orbit determination (POD) solutions, can then be updated in real time or more frequently.

Therefore, in this research effort, the problems of three carrier phase ambiguity resolution (TCAR) and position estimation (PE) are generalised as real time GNSS data processing problems for a continuously observing network of any large scales. Table 1 gives the summary of local, regional and global scale services being currently or potentially enabled by dual-frequency and triple-frequency carrier phase ambiguity resolutions. The shaded areas show the new services that TCAR technology can enable while the non-shaded areas indicate the current services that dual-frequency AR and PE can enable. In the triple frequency scenarios, AR and PE problems can be generalised as GNSS real time data processing problems using continuous observations from a network of continuously operating receivers distributed over any scales, i.e. local, regional and global scales.

Based on the generalisation of AR and PE problems, the rest of the paper is organised as follows. Section 2 presents a general formation of TCAR models, based on (i) geometry-free models; (ii) geometry-based models; and (iii) geometry-constrained models used along with geometry-based and geometry-free models and state transition equations for both real-value state parameters and floating ambiguity parameters. Section 3 provides all the estimation equations for the generalised models, which can cover the cases of various specific AR and PE problems and models. Some important long-distance AR performance benefits and impacts on wide area GNSS technology and applications will be outlined in Section 4. In the final section, the major findings of the paper are summarised.

Table 1 Summary of local, regional and global scale GNSS services, enabled by dual-frequency and triple frequency carrier phase ambiguity resolutions

Observation	Local scale: in terms of inter-station distance of typically up to 100 km		Regional scale in term of inter-station distance of tens to hundreds of km		Global scale: in terms of inter-station distance of hundreds to thousands of km	
Single-epoch observations (eg 1 second)	Single-base RTK (<20km)	Network-RTK	TCAR-based single-base RTK-decimetre	TCAR-based network-RTK, -centimetre	Global differential positioning	TCAR-based global RTK Decimetre
Multiple-epoch (observations (eg a few to tens of minutes)	Single-base RTK (<20km)	Network-RTK	TCAR-based single-base RTK-centimetre	TCAR-based network-RTK -centimetre	Precise Point Positioning-kinematic	TCAR-based global-RTK centimetre
Long arc observations (hours to days)	Baseline/network relative positioning-static		Baseline relative positioning and network-based PE and scientific applications		Precise orbit determination (POD) and scientific services	
Continuous observations	Generalised TCAR and PE for improved RTK services		Generalised TCAR and PE for real time GNSS services		Generalised TCAR for real time POD, PE, ZTD and scientific services	

2. General Formation of TCAR Problems

2.1 General models

We begin with the observation equations for the double-differenced (DD) phase and code measurements in meters,

$$\Delta\phi_i = \Delta\rho + \Delta\delta_{orb} + \Delta\delta_{tro} - \Delta\delta I_i - \lambda_i \Delta N_i + \varepsilon_{\Delta\phi_i} \tag{1}$$

and

$$\Delta P_i = \Delta\rho + \Delta\delta_{orb} + \Delta\delta_{tro} + \Delta\delta I_i + \varepsilon_{\Delta P_i} \tag{2}$$

In Eqs. (1) and (2), the symbol "Δ" represents the DD operation to the term immediately right; $\Delta\phi_i$ is the DD phase measurements at the ith frequency in meters, and ΔP_i is the DD code measurements at the ith frequency; the symbol Δ is the DD geometric distance, and $\Delta\delta_{orb}$, Δ_{tro} and ΔI_i are the DD satellite orbital error, DD tropospheric delay and DD ionospheric biases respectively, in meters.

In general, one can assume that each of the terms of the right-hand side of (1-2) is a function of one set of unknown state parameters or vectors, alongside the stochastic assumptions for the last term of the noise. As a result, (1) and (2) can be written as

$$\begin{aligned}\Delta\phi_i &= \Delta\rho(\mathbf{x}_1) + \Delta\delta_{orb}(\mathbf{x}_2) + \Delta\delta_{tro}(\mathbf{x}_3) \\ &\quad - \Delta\delta I_i(\mathbf{x}_4) - \Lambda_i(\mathbf{x}_5) + \varepsilon_{\Delta\phi_i}\end{aligned} \tag{3}$$

$$\Delta P_i = \Delta\rho(\mathbf{x}_1) + \Delta\delta_{orb}(\mathbf{x}_2) + \Delta\delta_{tro}(\mathbf{x}_3) + \Delta\delta I_i(\mathbf{x}_4) + \varepsilon_{\Delta P_i} \tag{4}$$

Given the number of rover receivers/baselines and number of satellites in view for (3) and (4) at each epoch, $\mathbf{x}_1$ is the user-specific state vector, $\mathbf{x}_2$ is the GNSS satellite-specific state vector for all satellites in view; $\mathbf{x}_3$ is the station-specific tropospheric vector; $\mathbf{x}_4$ is the 2D or 3D model parameters of DD ionospheric bias at the L1 carrier; $\mathbf{x}_5$ is the wavelength-specific ambiguity parameter vector of the DD phase measurements. It is assumed that independence between these parameters $\mathbf{x}_i$ (i=1,2,…,5) is maintained to ensure the solvability of (3) and (4). Here, it is emphasized that the ambiguity and ionosphere delay are dependent if a DD ionospheric parameter is set directly for each line of sight. For independent parameterisation under the multiple frequency cases between DD ambiguities and ionospheric biases, we notice the result by Odijk (2003). Basically, the ionosphere-free (IF) measurements are essentially employed for precise positioning and the actual ionospheric estimation cannot be achieved. Instead, it is suggested that the 2D or 3D ionospheric model parameters could be estimated instead to recover the actual ionospheric bias in the generalized model.

Setting the state vector,

$$\mathbf{x}_i = \mathbf{x}_i^0 + \delta\mathbf{x}_i, \quad i=1,2,\cdots,5 \tag{5}$$

with $\mathbf{x}_5^0 = 0$, one can obtain the computed DD geometric range

$$\Delta\rho^0 = \Delta\rho\left(\mathbf{x}_1^0\right) + \Delta\delta_{orb}\left(\mathbf{x}_2^0\right) + \Delta\delta_{tro}\left(\mathbf{x}_3^0\right) - \Delta\delta I_i\left(\mathbf{x}_4^0\right) \tag{6}$$

which is the same for all phase measurements and for all code measurements except the opposite sign for the ionospheric bias.

For convenience, we introduce the following vectors or variables,

$$\left.\begin{aligned} \mathbf{\Phi} &= \left(\Delta\phi_1 \quad \Delta\phi_2 \quad \Delta\phi_5\right)^T \\ \mathbf{P} &= \left(\Delta P_1 \quad \Delta P_2 \quad \Delta P_5\right)^T \\ \mathbf{N} &= \left(\Delta N_1 \quad \Delta N_2 \quad \Delta N_5\right)^T \\ \mathbf{\Lambda} &= \mathrm{diag}\left(\lambda_1 \quad \lambda_2 \quad \lambda_5\right) \end{aligned}\right\} \tag{7}$$

$$\left.\begin{aligned} \mathbf{Z}_{(i,j,k)} &= \frac{1}{i\cdot f_1 + j\cdot f_2 + k\cdot f_5}\left(i\cdot f_1 \quad j\cdot f_2 \quad k\cdot f_5\right) \\ \mathbf{z}_{(i,j,k)} &= \left(i \quad j \quad k\right) \end{aligned}\right\} \tag{8}$$

and

$$\left.\begin{aligned} \Delta N_{(i,j,k)} &= \mathbf{z}_{(i,,j,k)}\mathbf{N} \\ \lambda_{(i,j,k)} &= \frac{\lambda_1\lambda_2\lambda_5}{i\cdot\lambda_2\lambda_5 + j\cdot\lambda_1\lambda_5 + k\cdot\lambda_1\lambda_2} \end{aligned}\right\} \tag{9}$$

$$\tilde{\mathbf{\Phi}} = \mathbf{\Phi} + \mathbf{\Lambda}\mathbf{N} \tag{10}$$

In what follows, we examine the general expressions of various TCAR models.

2.2 Geometry-based TCAR models

Any geometry-based observables can be alternatively represented by the following linear transformations,

$$\left.\begin{aligned} \Delta P_{(i,j,k)} &= \mathbf{Z}_{(i,j,k)}\mathbf{P} \\ \Delta\phi_{(i,j,k)} &= \mathbf{Z}_{(i,j,k)}\mathbf{\Phi} \end{aligned}\right\} \tag{11}$$

As shown in Feng (2008) using the (11) instead of (1) and (2) allows for easier and more reliable AR in combined or separate steps, including the determination of the extra-widelane (EWL) ambiguity $\Delta N_{(0,1,-1)}$ with the geometry-free model, see e.g, Eq. (44), the second EWL integer ambiguity $\Delta N_{(1,-6,5)}$ with geometry-based models and the third ambiguity $\Delta N_{(1,0,0)}$ with two phase measurements.

Any ionosphere-free observables can also be expressed as linear transformation. For instance, code and phase IF measurements in the GPS L1 and L2 frequency case are given as

$$\left.\begin{aligned} \Delta P_{IF} &= \mathbf{Z}_{(77,-60,0)}\mathbf{P} \\ \Delta\phi_{IF} &= \mathbf{Z}_{(77,-60,0)}\mathbf{\Phi} \end{aligned}\right\} \tag{12}$$

The DD phase ionospheric delay with respect to the L1 carrier can be estimated as

$$\Delta\delta I_1 = \frac{f_2^2}{f_1^2 - f_2^2}\left(\mathbf{Z}_{(1,0,0)} - \mathbf{Z}_{(0,1,0)}\right)\tilde{\mathbf{\Phi}} \tag{13a}$$

or

$$\Delta\delta I_1 = \frac{f_2 f_5}{f_1(f_2 - f_5)}\left(\mathbf{Z}_{(1,0,-1)} - \mathbf{Z}_{(1,-1,0)}\right)\tilde{\mathbf{\Phi}} \tag{13b}$$

2.3 Geometry-free TCAR models

The general geometry-free TCAR models as given in Feng and Rizos (2009) can be formed as the linear combinations between virtual code and phase measurements,

$$\Delta P_{(l,m,n)} - \Delta\phi_{(i,j,k)} = \left[\mathbf{Z}_{(l,m,n)} \quad -\mathbf{Z}_{(i,j,k)}\right]\begin{bmatrix}\mathbf{P} \\ \mathbf{\Phi}\end{bmatrix} \tag{14}$$

and between two phase measurements as,

$$\Delta\tilde{\phi}_{(l,m,n)} - \Delta\phi_{(i,j,k)} = \mathbf{Z}_{(l,m,n)}\mathbf{\Phi} + \lambda_{(l,m,n)}N_{(l,m,n)} - \mathbf{Z}_{(i,j,k)}\mathbf{\Phi} \tag{15}$$

where the ambiguity $\Delta N_{(l,m,n)}$ has been primarily known; the subscripts (l,m,n) and (i,j,k) are used to represent two different integer sets in the formulae (14) and (15).

In both (14) and (15), the geometry-term, orbital error and tropospheric error or their related state parameters are cancelled; the effect of the ionospheric term can be reduced, thus allowing for direct and reliable estimation of two ambiguities. But it would take a much longer time span of averaging to correctly fix the third integer parameter $\Delta N_{(i,j,k)}$ due to the effects of enlarged phase noises in (15).

2.4 Geometric constraints

The equations (3) and (4) form a general observational model for a network of receivers with a certain number of satellites commonly in view. For a single epoch or over a short-time span, the (3) and (4) is an under-determined or severely ill-conditioned linear equation. To resolve the ambiguity parameters in (3) over a baseline or network with a short-observational span, the additional prior knowledge for some parameters should be introduced as much as possible. A general strategy is to impose the state constraint equations to the ith set of parameters $\mathbf{x}_i$ as,

$$\mathbf{A}_i\delta\mathbf{x}_i = \mathbf{w}_i \tag{16}$$

where $\mathbf{A}_i$ is the r-by-u coefficient matrix with u-by-1 state vector $\mathbf{x}_i$, $\mathbf{w}_i$ is the r-by-1 constant vector. The different constraint is characterized by its coefficient matrix $\mathbf{A}_i$ and constant vector $\mathbf{w}_i$. For instance, one can usually assume the use of the precise GNSS orbits solutions or assume a sufficient short baseline, the satellite specific parameters $\mathbf{x}_2$ is removed. Accordingly, matrix $\mathbf{A}_2$ and vector $\mathbf{w}_2$ in constraint equations are specialized as;

$$\mathbf{A}_2 = \mathbf{I},\quad \mathbf{w}_2 = \mathbf{0} \tag{17}$$

In the network-based process, all the station coordinates are precisely known, and it implies that the constraint equations introduced for $\mathbf{x}_1$ are:

$$\mathbf{A}_1 = \mathbf{I},\quad \mathbf{w}_1 = \mathbf{0} \tag{18}$$

More types of geometry constraints may include baseline length constraints, or horizontal and vertical coordinate component constraints respectively, as found in (Li & Shen 2009).

In addition, Li and Shen (2009) also gave a general constraint model for integer ambiguities, namely, for the parameter $\mathbf{x}_5$. For example, the constraint amongst integer ambiguities was given based on the fact there are just three DD ambiguities are independent in the case of single baseline with epochwise solution. In this situation, the constraint equations can be generally formulized as,

$$\mathbf{A}_5^1\mathbf{x}_5^1 + \mathbf{A}_5^2\mathbf{x}_5^2 = \mathbf{w}_5 \tag{19}$$

where $\mathbf{x}_5^1$ includes three ambiguities and $\mathbf{x}_5^2$ the rest ones. For the detailed information about coefficient matrix and constant vector, one is referred to Li and Shen (2009).

To sum up, the equations (3-4) and (16) give a complete and general formation of TCAR models for single or multiple epochs over which all the parameters are considered remaining unchanged. Any geometry-free and geometry-based TCAR problems can be derived from linear transformations of these fundamental code and phase observables for all the DD pairs and/or different constraint equations (16).

2.5 Parameterisations and linearization

For the convenient expression of the following context, we separate the parameters into two categories $\mathbf{x}_a$ and $\mathbf{x}_b$, and all real parameters are classified into $\mathbf{x}_a$ and all integer parameters (namely, ambiguities) into $\mathbf{x}_b$. Without loss of generality, it is assumed that independent parameterisation of the (3) and (4) are achievable. Performing linearisation of the equation system of (3), (4) and (16) with respect to the nominal value of all parameters in (5) leads to the overall linear observation equations and statistical model for the noise term,

$$\left.\begin{array}{l}\mathbf{L} = \mathbf{A}\delta\mathbf{x}_a + \mathbf{B}\delta\mathbf{x}_b + \boldsymbol{\varepsilon}\\ \mathrm{E}(\boldsymbol{\varepsilon}) = \mathbf{0},\ \mathrm{cov}(\boldsymbol{\varepsilon}) = \mathbf{Q}_\varepsilon\end{array}\right\} \tag{20}$$

where the components of the vector L. If for exemplar purposes one considers the user baseline $\mathbf{x}_1$, tropospheric and ionospheric parameters $\mathbf{x}_3$ and $\mathbf{x}_4$ only, the matrices A and B are expressed as for each satellite-receiver DD pair without are:

$$\mathbf{L} = \begin{pmatrix}\Delta P_1 - \Delta\rho^0\\ \Delta P_2 - \Delta\rho^0\\ \Delta P_5 - \Delta\rho^0\\ \Delta\phi_1 - \Delta\rho^0\\ \Delta\phi_2 - \Delta\rho^0\\ \Delta\phi_5 - \Delta\rho^0\end{pmatrix}, A = \begin{bmatrix} a_1 & a_2 & a_3 & a_4 & a_5\\ a_1 & a_2 & a_3 & a_4 & f_1^2/f_2^2 a_5\\ a_1 & a_2 & a_3 & a_4 & f_1^2/f_5^2 a_5\\ a_1 & a_2 & a_3 & a_4 & -a_5\\ a_1 & a_2 & a_3 & a_4 & -f_1^2/f_2^2 a_5\\ a_1 & a_2 & a_3 & a_4 & -f_1^2/f_5^2 a_5\end{bmatrix},$$

$$\mathbf{B} = \begin{pmatrix}0&0&0\\0&0&0\\0&0&0\\ \lambda_1&0&0\\0&\lambda_2&0\\0&0&\lambda_5\end{pmatrix},\ \boldsymbol{\varepsilon} = \begin{pmatrix}\Delta\varepsilon_{P_1}\\ \Delta\varepsilon_{P_2}\\ \Delta\varepsilon_{P_5}\\ \Delta\varepsilon_{\phi_1}\\ \Delta\varepsilon_{\phi_2}\\ \Delta\varepsilon_{\phi_5}\end{pmatrix};$$

where a_i are the partial derivation with respect to the initial values of the state parameter vector $\mathbf{x}_1$, $\mathbf{x}_3$ and $\mathbf{x}_4$ as specified for (3) and (4):

$$a_1 = \frac{\partial\Delta\rho}{\partial x}|_{x=x^0}, a_2 = \frac{\partial\Delta\rho}{\partial y}|_{x=x^0}, a_3 = \frac{\partial\Delta\rho}{\partial z}|_{x=x^0},$$

$$a_4 = \frac{\Delta\rho}{\partial\mathbf{x}_3}|_{x=x^0} = (m^p - m^q),\ a_5 = \frac{\Delta\rho}{\partial\mathbf{x}_4}|_{x=x^0} = -1$$

where $\mathbf{x}_3$ is a relative ZTD parameter for a single baseline; m^q and m^p are Niell's wet mapping function (Niell 1996) for satellite p and q. For example, m^p is expressed as:

$$m^p = \frac{1 + \dfrac{d}{1 + e/(1+g)}}{\sin\theta^p + \dfrac{d}{\sin\theta^p + e/(\sin\theta^p + g)}}$$

where θ^p is the average elevation angles of the two stations for satellites p; d, e, g are the coefficients and interpolated from tabular data, see e.g., Leick (2004).

Formation of the variance matrix Q_ε is naturally based on variance-covariance propagation from the original code and phase vectors.

The integer state vector $\mathbf{x}_b$ is equal to the integer vector $\mathbf{x}_5$. Therefore, the alternative formulation of constraint equations (16) can be separately expressed as,

$$\left.\begin{array}{l}\mathbf{C}_a\delta\mathbf{x}_a = \mathbf{w}_a\\ \mathbf{C}_b\delta\mathbf{x}_b = \mathbf{w}_b\end{array}\right\} \tag{21}$$

There are a total of up to $6r\times(s-1)$ DD measurements for each epoch and a network of (r+1) receivers and s satellites in view, using the same notations.

2.6 State transition equations

We now consider the more general case where the state vectors may vary from time to time over the whole observation period and the information of previous epochs can be accumulatively used to update the current real-valued states and floating solutions of the ambiguities. To reflect the state dynamics, we rewrite the Eq(20) with time index, k for t_k:

$$\mathbf{L}(k)=\mathbf{A}(k)\delta\mathbf{x}_a(k)+\mathbf{B}\delta\mathbf{x}_b(k)+\boldsymbol{\varepsilon}(k) \tag{22}$$

then introduce the state equations for real-valued state vector $\delta\mathbf{x}_a$,

$$\delta\mathbf{x}_a(k)=\boldsymbol{\Psi}_a(k,k-1)\delta\mathbf{x}_a(k-1)+\mathbf{u}(k-1) \tag{23a}$$

But the state equations for the integer state vector $\delta\mathbf{x}_b$ is applicable only for transition of the floating ambiguity solutions without a dynamic noise term,

$$\delta\mathbf{x}_b(k)=\boldsymbol{\Psi}_b(k,k-1)\delta\mathbf{x}_b(k-1) \tag{23b}$$

In Eqs.(23a, b), $\boldsymbol{\Psi}_a(k,k-1)$ and $\boldsymbol{\Psi}_b(k,k-1)$ are known as state transition matrices for propagating the prior state into the current one; k denotes the time epoch t_k; $\mathbf{u}$ is the dynamic noise vectors of the state $\mathbf{x}_a$. The stochastic statistic quantities for the observation vector $\boldsymbol{\varepsilon}$(k) in (22) and $\mathbf{u}$(k-1) in (23a) are specified as follows,

$$\left.\begin{aligned}E(\boldsymbol{\varepsilon}(k))=\mathbf{0},\operatorname{cov}(\boldsymbol{\varepsilon}(k))=\mathbf{Q}_{\boldsymbol{\varepsilon}(k)}\\ E(\mathbf{u}(k))=\mathbf{0},\operatorname{cov}(\mathbf{u}(k))=\mathbf{Q}_{\mathbf{u}(k)}\end{aligned}\right\} \tag{24}$$

Now we suppose that the state constraint equations applicable to the current epoch can be added as follows,

$$\left.\begin{aligned}\mathbf{C}_a\delta\mathbf{x}_a(k)=\mathbf{w}_a(k)\\ \mathbf{C}_b\delta\mathbf{x}_b(k)=\mathbf{w}_b(k)\end{aligned}\right\} \tag{25}$$

Eqs.(22-25) represent the general formulation of TCAR models, considering both state constraints and state dynamics or time variations of state vectors. Comments about state transitions equations for different state vectors are made in order:

In the traditional kinematic positioning case where the user states are independent from epoch to epoch, the user state vector $\mathbf{x}_1$ is a 3×1 positional vector for each baseline. For network-based data analysis, the user state vector should comprise a 6×1 positional and velocity vector for each station or baseline, in order to consider the effects of station variations over long distances.

The satellite state vector $\mathbf{x}_2$ is considered only in the regional or global network case. The state parameters of each satellite may include 3 positional parameters such along-track, radius and across-track components for short-arc; 6 orbital elements or 9 to 15 orbital and physical parameters, with the choices depending on the network scale, the filter methods and treatment of satellite dynamics, referring to the common strategies in GPS precise orbital determination systems such as Bernese, GAMIT software systems.

The zenith tropospheric delay (ZTD) state vector $\mathbf{x}_3$ may contain one vertical component and/or two gradient parameters for each station or relative ZTD for a baseline as shown in previously, depending on the scale of the network or baseline. In general, a random walk model may be used to propagate these state parameters from one epoch to another. However,

The initial state vector for the DD ionospheric state vector $\mathbf{x}_4$ is estimable with (13a) or (13b), which may be propagated with a polynomial function from epoch to epoch (Feng and Rizos, 2009).

The transition matrix for the ambiguity vector $\mathbf{x}_5$ is used to transit DD ambiguities from one set of DD matches to another, if there are any changes such as reference stations or satellites. Otherwise, the transition matrix would remain as an identity matrix.

The user state dynamic noise terms in general may be obtainable from statistics knowledge and experiences, although a conservative treatment, such setting them to zeros, may be applied in most of computational situations.

3 Generalised TCAR Solutions

3.1 Generalised TCAR equations

For the generalised observational equations (22), state equations (23) and state constraint equations (25), the standard Kalman filter has to be modified to incorporate the state constraints in the filter, which is called constrained Kalman filter or Kalman filter with state constraints in literatures for control theory and applications. There are many examples of state-constrained systems in engineering applications, including camera tracking (Julier, et al, 2007), fault diagnosis (Simon, et al, 2006), vision-based systems (Porrill, 1988), target tracking (Wang et al, 2002], robotics (Spong, et al, 2005). The number of algorithms for constrained state estimation has been overwhelming, depending how the problem is viewed from different perspectives. A linear relationship between states implies a reduction of the state dimension, for instance, without considering satellite orbits states. Constrained Kalman Filtering can be viewed as a constrained likelihood maximisation problem or a constrained least squares, thus the method is called projection approach (Simon, et al, 2002; Teixeira et al, 2009). Therefore, we give the generalised TCAR solutions following the constraint least-square estimation, comprising 4 steps as follows.

Step 1: Prediction of the state estimator

Given an estimate of the state vector $\delta\hat{\mathbf{x}}(k-1)$ at the (k-1)th epoch, the state vector at any later time t_k can be predicted with use of the transition matrix. This predicted estimator of $\delta\mathbf{x}(k)$ here denoted $\delta\tilde{\mathbf{x}}(k)$ is given by,

$$\left.\begin{aligned}\delta\tilde{\mathbf{x}}_a(k) &= \mathbf{\Psi}_a(k,k-1)\delta\hat{\mathbf{x}}_a(k-1)\\ \delta\tilde{\mathbf{x}}_b(k) &= \mathbf{\Psi}_b(k,k-1)\delta\hat{\mathbf{x}}_b(k-1)\end{aligned}\right\} \quad (26)$$

Similarly the prediction of the covariance matrix of the predicted estimator of $\delta\tilde{\mathbf{x}}(k)$ is given by

$$\left.\begin{aligned}\mathbf{Q}_{\delta\tilde{\mathbf{x}}_a(k)} &= \mathbf{\Psi}_a(k,k-1)\mathbf{Q}_{\delta\hat{\mathbf{x}}_a(k-1)}\mathbf{\Psi}_a^T(k,k-1)+\mathbf{Q}_{\mathbf{U}(k-1)}\\ \mathbf{Q}_{\delta\tilde{\mathbf{x}}_b(k)} &= \mathbf{\Psi}_b(k,k-1)\mathbf{Q}_{\delta\hat{\mathbf{x}}_b(k-1)}\mathbf{\Psi}_b^T(k,k-1)\\ \mathbf{Q}_{\delta\tilde{\mathbf{x}}_{ab}(k)} = \mathbf{Q}_{\delta\tilde{\mathbf{x}}_{ab}(k)}^T &= \mathbf{\Psi}_a(k,k-1)\mathbf{Q}_{\delta\hat{\mathbf{x}}_{ab}(k-1)}\mathbf{\Psi}_b^T(k,k-1)\end{aligned}\right\} \quad (27)$$

Referring to the definition of the measurement epoch in Section 2, we must notice that the time intervals between epoch t_k and t_{k-1} can be different in different data analysis problems, such as 1 second in real time kinematic positioning and 5 minutes in precise orbit determination. In general, we can assume that $\mathbf{L}(k)$ contains all the measurements over the interval propagated to the time t_k via the state transition matrix.

Step 2: Standard Kalman filter solutions

Given the observation $\mathbf{L}(k)$ at the time t_k with associated observational covariance matrix $\mathbf{Q}_{\varepsilon(k)}$, the standard filter estimates of the state vector $\delta\mathbf{x}(k)$ with considering the predicted state estimator $\delta\tilde{\mathbf{x}}(k)$ is obtained from the following equations,

$$\begin{bmatrix}\delta\hat{x}_a(k)\\ \delta\hat{x}_b(k)\end{bmatrix} = \begin{bmatrix}\delta\tilde{x}_a(k)\\ \delta\tilde{x}_b(k)\end{bmatrix} + \begin{bmatrix}G_{aa} & G_{ab}\\ G_{ba} & G_{bb}\end{bmatrix}\begin{bmatrix}u_a\\ u_b\end{bmatrix} \quad (28)$$

where

$$\begin{bmatrix}\mathbf{G}_{aa} & \mathbf{G}_{ab}\\ \mathbf{G}_{ba} & \mathbf{G}_{bb}\end{bmatrix} = \begin{bmatrix}\mathbf{A}^T(k)\mathbf{Q}_{\varepsilon(k)}^{-1}\mathbf{A}(k)+\mathbf{R}_{\delta\tilde{\mathbf{x}}_a(k)} & \mathbf{A}^T(k)\mathbf{Q}_{\varepsilon(k)}^{-1}\mathbf{B}(k)+\mathbf{R}_{\delta\tilde{\mathbf{x}}_{ab}(k)}\\ \mathbf{B}^T(k)\mathbf{Q}_{\varepsilon(k)}^{-1}\mathbf{A}(k)+\mathbf{R}_{\delta\tilde{\mathbf{x}}_{ba}(k)} & \mathbf{B}^T(k)\mathbf{Q}_{\varepsilon(k)}^{-1}\mathbf{B}(k)+\mathbf{R}_{\delta\tilde{\mathbf{x}}_b(k)}\end{bmatrix}^{-1} \quad (29)$$

$$\begin{bmatrix}\mathbf{u}_a\\ \mathbf{u}_b\end{bmatrix} = \begin{bmatrix}\mathbf{A}^T(k)\mathbf{Q}_{\varepsilon(k)}^{-1}\left(\mathbf{L}(k)-\mathbf{A}(k)\delta\tilde{\mathbf{x}}_a(k)-\mathbf{B}(k)\delta\tilde{\mathbf{x}}_b(k)\right)\\ \mathbf{B}^T(k)\mathbf{Q}_{\varepsilon(k)}^{-1}\left(\mathbf{L}(k)-\mathbf{A}(k)\delta\tilde{\mathbf{x}}_a(k)-\mathbf{B}(k)\delta\tilde{\mathbf{x}}_b(k)\right)\end{bmatrix} \quad (30)$$

while

$$\begin{bmatrix}\mathbf{R}_{\delta\tilde{\mathbf{x}}_a(k)} & \mathbf{R}_{\delta\tilde{\mathbf{x}}_{ab}(k)}\\ \mathbf{R}_{\delta\tilde{\mathbf{x}}_{ba}(k)} & \mathbf{R}_{\delta\tilde{\mathbf{x}}_b(k)}\end{bmatrix} = \begin{bmatrix}\mathbf{Q}_{\delta\tilde{\mathbf{x}}_a(k)} & \mathbf{Q}_{\delta\tilde{\mathbf{x}}_{ab}(k)}\\ \mathbf{Q}_{\delta\tilde{\mathbf{x}}_{ba}(k)} & \mathbf{Q}_{\delta\tilde{\mathbf{x}}_b(k)}\end{bmatrix}^{-1}$$

Eq (29) is also the variance and covariance matrix of the standard filter solution (28).

Step 3: Constrained Kalman Filter Solutions

The constrained filtering is to project the unconstrained estimate of the Kalman filter $\delta\hat{x}(k)$ onto the constraint surface. The constrained estimate can therefore be obtained by satisfying the following criterion:

$$(\delta x_a - \delta\hat{x}_a)^T G_{aa}^{-1}(\delta x_a - \delta\hat{x}_a) = \min \quad (31)$$

Such that

$$C_a\delta x_a = w_a$$

The constrained *a priori* estimate is based on the unconstrained estimate so that the constrained filter is

$$\begin{bmatrix}\delta\breve{x}_a\\ \delta\breve{x}_b\end{bmatrix} = \begin{bmatrix}\delta\hat{x}_a\\ \delta\hat{x}_b\end{bmatrix} - \begin{bmatrix}G_{aa}C_a^T(C_aG_{aa}C_a^T)[w_a - C_a\delta\hat{x}_a]\\ G_{ba}C_a^T(C_aG_{aa}C_a^T)[w_a - C_a\delta\hat{x}_a]\end{bmatrix} \quad (32)$$

The covariance matrix is then expressed as:

$$\begin{bmatrix}\mathbf{Q}_{\delta\breve{\mathbf{x}}_a(k)} & \mathbf{Q}_{\delta\breve{\mathbf{x}}_{ab}(k)}\\ \mathbf{Q}_{\delta\breve{\mathbf{x}}_{ba}(k)} & \mathbf{Q}_{\delta\breve{\mathbf{x}}_b(k)}\end{bmatrix} = \begin{bmatrix}\mathbf{G}_{aa}\left(\mathbf{I}_a - \mathbf{C}_a^T\left(\mathbf{C}_a\mathbf{G}_{aa}\mathbf{C}_a^T\right)^{-1}\mathbf{C}_a\mathbf{G}_{aa}\right) & \mathbf{G}_{ab} - \mathbf{G}_{aa}\mathbf{C}_a^T\left(\mathbf{C}_a\mathbf{G}_{aa}\mathbf{C}_a^T\right)^{-1}\mathbf{C}_a\mathbf{G}_{ab}\\ \mathbf{G}_{ba} - \mathbf{G}_{ba}\mathbf{C}_a^T\left(\mathbf{C}_a\mathbf{G}_{aa}\mathbf{C}_a^T\right)^{-1}\mathbf{C}_a\mathbf{G}_{aa} & \mathbf{G}_{bb} - \mathbf{G}_{ba}\mathbf{C}_a^T\left(\mathbf{C}_a\mathbf{G}_{aa}\mathbf{C}_a^T\right)^{-1}\mathbf{C}_a\mathbf{G}_{ab}\end{bmatrix} \quad (33)$$

If the constrained *a priori* estimate is based on the constrained estimate then the time update (26) should be rewritten as [14]

$$\begin{aligned}\delta\tilde{x}_a(k) &= \Psi_a(k,k-1)\delta\breve{x}_a(k-1)\\ \delta\tilde{x}_b(k) &= \Psi_b(k,k-1)\delta\breve{x}_b(k-1)\end{aligned} \quad (34)$$

However, if the same state constraints for each epoch, the time update (34) is not suggested.

Step 4: Integer ambiguity search

Step 3 finally results in the float solution for the integer state vector $\delta\hat{\mathbf{x}}_b(k)$ in (32) and the covariance matrix (33). The integer least squares criterion is now used for integer ambiguity search due to the ambiguity property of discrete as,

$$\min_{\mathbf{z}}: \Phi = \left(\mathbf{z} - \delta\hat{\mathbf{x}}_b\right)^T \mathbf{Q}_{\delta\breve{\mathbf{x}}_b(k)}^{-1}\left(\mathbf{z} - \delta\hat{\mathbf{x}}_b\right) \quad (35)$$

In the ambiguity search procedure, application of the constraint equation amongst the ambiguities will shrink the volume of search ellipsoid and then improve the integer search efficiency. Referring to Li and Shen (2007), if the constraint equations (19) are used, we can transform the search of $\mathbf{x}_5$ into $\mathbf{x}_5^1$, the search dimension is reduced to 3 for whatever the dimension of $\mathbf{x}_5$ would be. In addition, they introduced the constraint equations amongst the ambiguities at the different frequencies as well to enhance the search speed. As far as the search algorithm is concerned, we refer to the LAMBDA (least squares ambiguity decorrelation adjustment) method of Teunissen (1994) which basically employs a decorrelation technique to minimize the correlation of ambiguities has been commonly used for AR with single or dual frequency GPS data in static or kinematic positioning scenarios.

3.2 Simplified TCAR solutions

Generalisation does not necessarily make the problems complicated. Instead it provides a uniform theoretical framework to cover various real time GNSS data processing scenarios. In fact, various TCAR-based real time positioning solutions belong to simplified versions of the generalised TCAR solutions given in Section 3.1. In this sub-section, we derive the TCAR solutions of several typical applications.

Geometry-based AR with single epoch measurements

In this case, the **G** matrix (30) and **u** vector (31) will be reduced to the following expressions,

$$\begin{bmatrix}\mathbf{G}_{aa} & \mathbf{G}_{ab}\\ \mathbf{G}_{ba} & \mathbf{G}_{bb}\end{bmatrix}=\begin{bmatrix}\mathbf{A}^T(k)\mathbf{Q}_{\varepsilon(k)}^{-1}\mathbf{A}(k) & \mathbf{A}^T(k)\mathbf{Q}_{\varepsilon(k)}^{-1}\mathbf{B}(k)\\ \mathbf{B}^T(k)\mathbf{Q}_{\varepsilon(k)}^{-1}\mathbf{A}(k) & \mathbf{B}^T(k)\mathbf{Q}_{\varepsilon(k)}^{-1}\mathbf{B}(k)\end{bmatrix}^{-1} \quad (36)$$

$$\begin{bmatrix}\mathbf{u}_a\\ \mathbf{u}_b\end{bmatrix}=\begin{bmatrix}\mathbf{A}^T(k)\mathbf{Q}_{\varepsilon(k)}^{-1}\mathbf{L}(k)\\ \mathbf{B}^T(k)\mathbf{Q}_{\varepsilon(k)}^{-1}\mathbf{L}(k)\end{bmatrix} \quad (37)$$

The solutions are similarly referred to (29) and (32). Without constraint equations, e.g., $\mathbf{C}_a=\mathbf{0}$, the final solution (33) is simply reduced to

$$\begin{bmatrix}\delta\hat{\mathbf{x}}_a(k)\\ \delta\hat{\mathbf{x}}_b(k)\end{bmatrix}=\begin{bmatrix}\mathbf{G}_{aa} & \mathbf{G}_{ab}\\ \mathbf{G}_{ba} & \mathbf{G}_{bb}\end{bmatrix}\begin{bmatrix}\mathbf{u}_a\\ \mathbf{u}_b\end{bmatrix} \quad (38)$$

and its variance-covariance matrix is,

$$\begin{bmatrix}\mathbf{Q}_{\delta\hat{x}_a(k)} & \mathbf{Q}_{\delta\hat{x}_{ab}(k)}\\ \mathbf{Q}_{\delta\hat{x}_{ba}(k)} & \mathbf{Q}_{\delta\hat{x}_b(k)}\end{bmatrix}=\begin{bmatrix}\mathbf{A}^T(k)\mathbf{Q}_{\varepsilon(k)}^{-1}\mathbf{A}(k) & \mathbf{A}^T(k)\mathbf{Q}_{\varepsilon(k)}^{-1}\mathbf{B}(k)\\ \mathbf{B}^T(k)\mathbf{Q}_{\varepsilon(k)}^{-1}\mathbf{A}(k) & \mathbf{B}^T(k)\mathbf{Q}_{\varepsilon(k)}^{-1}\mathbf{B}(k)\end{bmatrix}^{-1} \quad (39)$$

Geometry-free AR solution with measurements of a single epoch

Through the linear transformation from Eqs.(14) and (15) to Eq (20), we obtain the geometry-free model for AR,

$$L_s = B_s\delta x_b + \varepsilon_s \quad (40a)$$

where the transformed observation vector $\mathbf{L}_S=\mathbf{SL}$, the coefficient matrix $\mathbf{B}_s=\mathbf{SB}$ and transformed observation noise $\boldsymbol{\varepsilon}_S=\mathbf{S}\boldsymbol{\varepsilon}$ and then covariance matrix $\mathbf{Q}_{\varepsilon_S}=\mathbf{SQ}_{\varepsilon}\mathbf{S}^T$. The transformation matrix for the ionosphere-free example is given as follows (Li et al., 2010)

$$S=\begin{pmatrix}\frac{f_1}{f_1+f_2} & \frac{f_2}{f_1+f_2} & 0 & \frac{-f_1}{f_1-f_2} & \frac{f_2}{f_1-f_2} & 0\\ \frac{f_1}{f_1+f_5} & 0 & \frac{f_5}{f_1+f_5} & \frac{-f_1}{f_1-f_5} & 0 & \frac{f_5}{f_1-f_2}\\ 0 & 0 & 0 & b_1 & b_2 & b_3\end{pmatrix} \quad (40b)$$

where

$$b_1=\frac{\alpha_1 f_1}{f_1-f_2}+\frac{\alpha_2 f_1}{f_1-f_5}, b_2=\frac{-\alpha_1 f_2}{f_1-f_2}, b_3=\frac{\alpha_1 f_5 - f_1}{f_1-f_5},$$

with definitions of

$$\alpha_1=\frac{(f_1-f_5)f_2}{(f_2-f_5)f_5}, \alpha_2=1-\alpha_1$$

and **S** satisfying that $\mathbf{SA}=\mathbf{0}$. The ambiguity vector can be estimated,

$$\delta\hat{x}_b=(B_S^T Q_{\varepsilon s}B_S)^{-1}B_B^T Q_{\varepsilon s}L_s \quad (41)$$

Without considering the error correlation in L_s, the geometry-free and ionosphere-free float ambiguity solution can be further simplified as:

$$\delta\hat{x}_b=(B_S^T B_S)^{-1}B_S^T L_s$$

$$=\begin{pmatrix}\frac{\Delta P_{(1,1,0)}-\phi_{(1,-1,0)}}{\lambda_{(1,1,0)}} & \frac{\Delta P_{(1,1,0)}-\phi_{(1,-1,0)}}{\lambda_{(1,1,0)}} & \frac{\alpha_1\Delta\tilde{\phi}_{(1,1,0)}+\alpha_2\tilde{\phi}_{(1,0,-1)}-\phi_{(0,0,1)}}{\lambda_5}\end{pmatrix}^T \quad (42)$$

Geometry-free AR solution using measurements of multiple epochs

Considering measurements over multiple epochs, we introduce the time epoch index in the geometry-free model (40),

$$L_s(k)=B_s\delta x_b(k)+\varepsilon_s)k) \quad (43a)$$

the state transition equation,

$$\delta\mathbf{x}_b(k)=\delta\mathbf{x}_b(k-1) \quad (43b)$$

The matrix (29) now becomes

$$G_{bb}=[B_S^T Q_{\varepsilon s}^{-1}B_S+R_{\delta\tilde{x}_b(k)}]^{-1}=[\sum_{k=1}^{n}B_S^T Q_{\varepsilon_S}^{-1}B_S]^{-1} \quad (43c)$$

$$u_b=B_S^T Q_S^{-1}B_s[L_s(k)-B\delta\tilde{x}(k)] \quad (44a)$$

where $\delta\tilde{\mathbf{x}}_b(k)=\delta\hat{\mathbf{x}}_b(k-1)$. Thus we have the float solution:

$$\delta\hat{x}_b=\delta\tilde{x}_b+\mathbf{G}_{bb}\mathbf{u}_b \quad (44c)$$

Considering the diagonal nature of both $\mathbf{B_S}(k)$ and $\mathbf{G}_{bb}$ matrices and Ignoring the correlation in Ls, one can easily derive that the float solution of the $\delta\mathbf{x}_b(k)$ is actually the average observational vector $\mathbf{L_s}(k)$ over time divided by the corresponding wavelengths.

Simplified TCAR models

Simplification of TCAR problems leads to two results: (i) the observation equation can be simplified to include only a minimal number of state parameters, and (ii) a full AR problem for three-frequencies is decomposed into three sets of AR problems, and each set of ambiguity is resolved at a time, so that a complete TCAR problem is significantly reduced. For instance, TCAR with measurements of single epoch as defined by Eq. (20) can be completed with the following three separate steps:

Step 1 is the geometry-free determination of the EWL formed between the two closest L-band carrier measurements, directly from the two corresponding code measurements from (15),

$$\Delta P_{(0,1,1)}-\Delta\phi_{(0,1,-1)}=[\mathbf{Z}_{(0,1,1)} \quad -\mathbf{Z}_{(0,1,-1)}]\begin{bmatrix}\mathbf{P}\\ \mathbf{\Phi}\end{bmatrix}\approx\lambda_{(0,1,-1)}\Delta N_{(0,1,-1)} \quad (45)$$

The estimate of $\Delta N_{(0,1,-1)}$ is the corresponding float estimate rounded off to the nearest integer.

Step 2 forms the second EWL signal and resolves the integer ambiguity with a geometry-based estimator alone. The observation equation will be (Feng, 2008),

$$\begin{bmatrix} \Delta P_{IF} - \Delta\rho^0 \\ \Delta\phi_{(1,-6,5)} - \Delta\rho^0 \end{bmatrix} = \begin{bmatrix} \mathbf{A} & 0 \\ \mathbf{A} & -\lambda_{(1,-6,5)} \end{bmatrix} \begin{bmatrix} \delta\mathbf{x} \\ \Delta N_{(1,-6,5)} \end{bmatrix} + \begin{bmatrix} \varepsilon_{\Delta P_{IF}} \\ \varepsilon_{\Delta\phi_{(1,-6,5)}} \end{bmatrix} \quad (46a)$$

where the effect of the DD ionospheric bias with respect to L1 frequency is reduced to the factor of -0.0704 in $\Delta\phi_{(1,-6,5)}$ and the effects of the tropospheric errors are negligible with respect to the wavelength. In other words, both ionospheric and tropospheric parameters are set to zeros. It is important to note that the ionospheric-free code measurement ΔP_{IF} is normally very noisy. Over medium baselines where the ionospheric delay may be smaller than the level of code noise in ΔP_{IF}, the virtual codes $\Delta P_{(1,1,0)}$ can be used instead of ΔP_{IF} in (45) , resulting in the following equation.

$$\begin{bmatrix} \Delta P_{(1,1,0)} - \Delta\rho^0 \\ \Delta\phi_{(1,-6,5)} - \Delta\rho^0 \end{bmatrix} = \begin{bmatrix} \mathbf{A} & 0 \\ \mathbf{A} & -\lambda_{(1,-6,5)} \end{bmatrix} \begin{bmatrix} \delta\mathbf{x} \\ \Delta N_{(1,-6,5)} \end{bmatrix} + \begin{bmatrix} \varepsilon_{\Delta P_{(1,1,0)}} \\ \varepsilon_{\Delta\phi_{(1,-6,5)}} \end{bmatrix} \quad (46b)$$

Step 3 finds an independent observable, which is used together with a $\Delta P_{(1,1,0)}$ or refined WL to resolve the third ambiguity with geometry-based integer estimation and search algorithms, depending on the total noise levels of respective code and selected third observables. One can choose the following linear question to complete the AR process,

$$\begin{bmatrix} \Delta\tilde{\phi}_{(1,-1,0)} - \Delta\rho^0 \\ \Delta\phi_{(1,0,0)} - \Delta\rho^0 \end{bmatrix} = \begin{bmatrix} \mathbf{A} & 0 \\ \mathbf{A} & -\lambda_{(1,0,0)} \end{bmatrix} \begin{bmatrix} \delta\mathbf{x} \\ \Delta N_{(1,0,0)} \end{bmatrix} + \begin{bmatrix} \varepsilon_{\Delta\tilde{\phi}_{(1,-1,0)}} \\ \varepsilon_{\Delta\phi_{(1,0,0)}} \end{bmatrix} \quad (47)$$

Both ionospheric and tropospheric parameters should be considered for the medium to long baselines. The problem is the linear model (47) would become ill-posed. Li et al.(2009) suggested an extended GNSS ambiguity resolution with regularization criterion and constraints as a possible solution. One may notice that the third signal can also be determined with the geometry-free model (42).

4. Performance Benefits and Impacts

The theoretical analysis by Feng (2008) and Feng and Rizos (2009) and experimental analysis with semi-generated triple frequency data from long baselines (Feng and Li, 2008), Feng and Li, 2009, and Li et al 2009), have demonstrated a number of key performance TCAR benefits. The first key benefit of the additional frequency is that one can form two best extra-widelane virtual observables to allow for very easy and reliable determination of their ambiguities. The procedures include the rounding process for the first EWL observable and the LAMBDA process for the second EWL with its geometry-based model. The time to ambiguity fix is basically a single epoch in the both cases. Importantly, this performance can be achieved with very little distance constraints, as long as the base and rover receivers have sufficient satellites commonly in view. Secondly, with the two ambiguities-fixed EWLs or their derived WLs, the ionosphere-free WL phase can be obtained for position estimation without resolution of the third ambiguity. Experimental results have demonstrated the overall 3D RMS accuracy of 20 cm achievable with smoothing process over just 100 seconds (Feng and Li, 2009). The dominating error factor for this level of positioning is the residual tropospheric bias in DD phase measurements. With respect to dual-frequency based wide area differential positioning, a major benefit of using the third frequency signals is the reduction of convergence time for the decimetre RMS accuracy from a few tens of minutes to a few minutes. The user terminal can update the DD ionospheric biases to the accuracy of a few centimetres with the above two ambiguity resolvable WL observables from epoch to epoch. As a result, the above accuracy of the 20 cm can be maintained using the ionospheric estimations of previous epochs virtually all the times. Phase breaks for whatever reasons impose very little impact on the continuity of the solutions. The third benefit is that the 100% AR success rates of the third ambiguity has achieved via the simple averaging/smoothing process over a period of about 6 to 7 minutes. This result has significant performance potential for regional and global RTK and other real time GNSS applications, such as real time GNSS orbit determinations.

5. Concluding Remarks

This paper has contributed to generalisation of the problems of the TCAR and PE into real time GNSS data analysis problems with a continuously observing network on any scale. A general linear equation system has been presented that unifies geometry-free, geometry-based and geometry-constraint TCAR models and the state transition questions from time to time. Generalised TCAR solutions can inversely be simplified to the different integer solutions, such as geometry-free rounding and geometry-based LAMBDA solutions with measurements of single epoch or multiple epochs. Review of TCAR performance benefits based on the data analyses in some recent literatures have shown profound implications for the future GNSS development from both technology and application perspectives.

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BIOGRAPHY

Yanming Feng received his PhD degree in satellite geodesy from Wuhan Technical University of Surveying and Mapping (now part of Wuhan University), China. He is Associate Professor of Queensland University of Technology, Australia, responsible for teaching and research programs in the wireless communication and navigation theme. His research interests include dedicated short-range communications, satellite orbit determination, wide area GNSS positioning, multiple carrier ambiguity resolution and real time kinematic positioning and applications. He currently serves as the Editor-in-Chief for Journal of Global Positioning Systems