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Extended GNSS ambiguity resolution models with regularization criterion and constraints

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Abstract

This paper firstly presents an extended ambiguity resolution model that deals with an ill-posed problem and constraints between the estimated parameters. In the extended model, the regularization criterion is used instead of the traditional least squares in order to estimate the float ambiguities better. The existing models can be derived from the general model. Secondly, the paper examines the existing ambiguity searching methods from four aspects: exclusion of nuisance integer candidates based on the available integer constraints; integer rounding; integer bootstrapping and integer least squares estimations. Finally, an experiment is carried out to demonstrate the ambiguity resolution performance under the regularization criterion with different constraints.

Keywords GNSS, Ambiguity resolution, regularization, constraints

1 Introduction

The GNSS carrier phase observations can be used for various precise positioning applications, including real time kinematic positioning and atmospheric modelling services, depending on the integer ambiguities in the phase measurements being correctly fixed. The ambiguity resolution (AR) topic has been intensively investigated and a great number of methods proposed in the past two decades for single, dual and three frequency GNSS applications, with the contributions by Dong and Bock (1989); Frei and Beulter (1990); Hatch (1990); Teunissen (1993); Park et al. (1996); Shen and Li (2007); Li and Shen (2008); Feng and Rizos (2005); Feng (2008); Li et al. (2010) etc. Nevertheless it is still an open and critical problem attracting much research attentions (Verhagen 2004). In particular, with the availability of multiple frequency GNSS signals and more demanding requirements from users, the issues such as reliability of instantaneous AR, the efficiency of searching techniques for huge number of integer parameters due to multiple frequencies and multiple GNSS systems, become more critical (Feng 2008).

An AR method is identified to be effective from three aspects: (i) the difference between estimated float ambiguities and their true integers is sufficiently close and their covariance matrix can be decorrelated sufficiently to a diagonal matrix; (ii) the searching technique can efficiently pick up the integer candidates considered correct by excluding all nuisance candidates; (iii) the final integer ambiguities should be statistically evaluated to further avoid the incorrect AR solutions. Various AR methods have been developed to improve one or more of the three aspects mentioned above, while most of them have concentrated on the second aspects.

In fast GPS positioning, the normal equations associated to ambiguities and coordinate parameters is severely of ill-condition. The float ambiguities can largely deviate from their integers and their covariance matrices are highly correlated. In order to obtain a better set of float ambiguities that are more closer to their integers with a less correlated covariance matrix, Shen and Li (2007) introduce the regularization criterion instead of the traditional least squares (LS) such that the degree of the ill-condition of normal equations is significantly reduced, also see e.g., Li and Shen (2008). Furthermore, Li and Shen (2009) gave a general AR model with available constraints in the context of LS adjustment, and the quality of the float solution is also improved. As far as the efficient ambiguity searching is concerned, substantial contributions are referred to the decorrelation technique firstly proposed by Teunissen (1993). Afterwards, the decorrelation technique has been explored, including efforts by Liu et al. (1999), Grafarend (2000) and Xu (2001). It is noted that many researchers have worked on the validation of AR, see e.g., Frei and Beutler (1990), Euler and Schaffrin (1990), Han (1997), Teunissen (1999), Verhagen (2004), Xu (2006).

This paper presents an extended AR model that deals with an ill-posed problem and constraints among the estimated parameters. The extended model uses a regularization criterion instead of the traditional LS criterion, and the existing models can be derived from the general model. In section 3, the paper examines the existing ambiguity search methods from four aspects, exploring their relationship and difference with each other. In Sec 4, a numerical experiment is carried out to demonstrate the AR performance of regularization method with constraints. Finally, concluding remarks are given in Sec 5.

2 Generalized ambiguity searching criterion

2.1 Regularized ambiguity resolution with constraints

We start with the linearized double differenced (DD) observation model

$$\mathbf{v} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{z} - \mathbf{y}, \quad \mathrm{cov}(\mathbf{y}) = \sigma_0^2 \mathbf{P}^{-1} \tag{1}$$

where $\mathbf{y}$ and $\mathbf{v}$ are the ($n\times1$) vectors of DD GNSS observations and residuals; $\mathbf{A}$ and $\mathbf{B}$ are the ($n\times t_1$) and ($n\times t_2$) coefficient matrices of ($t_1\times1$) real-valued parameters $\mathbf{x}$ and ($t_2\times1$) integer parameters $\mathbf{z}$ with full column rank. $\sigma_0^2\mathbf{P}^{-1}$ is the covariance matrix of DD observations $\mathbf{y}$, where σ_0^2 is a prior variance of unit weight and $\mathbf{P}$ the weight matrix.

To improve AR reliability and also speed up the AR integer search, one shall make use of available constraints about the real valued and integer parameters. The generalized linear constraints are expressed as (Li and Shen 2009)

$$\mathbf{R}\mathbf{x} - \mathbf{w} = \mathbf{0} \tag{2}$$

and

$$\mathbf{S}\mathbf{z} - \mathbf{q} = \mathbf{0} \tag{3}$$

for real-valued and integer parameters, respectively. Here $\mathbf{R}$ and $\mathbf{S}$ are ($m_1\times t_1$) and ($m_2\times t_2$) coefficient matrices for constraints of real-valued and integer parameters with full row rank, and $\mathbf{w}$ and $\mathbf{q}$ are their corresponding ($m_1\times1$) and ($m_2\times1$) constant vectors.

Normally, we first solve the "float" ambiguities as real numbers regardless of their integer characteristics. In this step, all constraints for real-valued parameters can be used to enhance the float solutions. However, the constraints for integer parameters cannot in principle be used unless these integer constraints are also admissible with the float ambiguities. Moreover, in the rapid AR from several epochs or even on the fly, the normal equations are usually severely ill-posed. In this situation, the small errors in the observations do lead to the larger errors in the estimates. The regularization criterion is employed instead of the traditional LS criterion to stabilize the float solution (Shen and Li 2007). The objective function Φ of float solution based on the regularization criterion with available constraints for real-valued parameters is expressed as

$$\begin{aligned}\Phi = &\left(\mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{z} + \mathbf{y}\right)^T \mathbf{P} \left(\mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{z} - \mathbf{y}\right) \\ &+ 2\mathbf{k}^T\left(\mathbf{R}\mathbf{x} - \mathbf{w}\right) + \alpha \mathbf{z}^T\mathbf{z} = \min\end{aligned} \tag{4}$$

where α is the regularization parameter of a positive value. If this parameter is properly given, better float ambiguities with slight deviation to their actual integers and low correlation in the covariance matrix can be achieved than those from the LS criterion (Shen and Li 2007; Li and Shen 2008). Let the derivatives of the objective function with respect to the parameter $\mathbf{x}$, $\mathbf{z}$ and $\mathbf{k}$ equal to zeros, we obtain

$$\partial\Phi/\partial\mathbf{x}\Big|_{\hat{\mathbf{x}},\hat{\mathbf{z}},\hat{\mathbf{k}}} = 2\mathbf{A}^T\mathbf{P}\left(\mathbf{A}\hat{\mathbf{x}} + \mathbf{B}\hat{\mathbf{z}} - \mathbf{y}\right) + 2\mathbf{R}^T\hat{\mathbf{k}} = \mathbf{0} \tag{5a}$$

$$\partial\Phi/\partial\mathbf{z}\Big|_{\hat{\mathbf{x}},\hat{\mathbf{z}},\hat{\mathbf{k}}} = 2\mathbf{B}^T\mathbf{P}\left(\mathbf{A}\hat{\mathbf{x}} + \mathbf{B}\hat{\mathbf{z}} - \mathbf{y}\right) + 2\alpha\hat{\mathbf{z}} = \min \tag{5b}$$

$$\partial\Phi/\partial\mathbf{k}\Big|_{\hat{\mathbf{x}},\hat{\mathbf{z}},\hat{\mathbf{k}}} = 2\left(\mathbf{R}\hat{\mathbf{x}} - \mathbf{w}\right) = \mathbf{0} \tag{5c}$$

where the variables with hats denote the LS estimates. Then the normal equations are

$$\begin{pmatrix} \mathbf{N}_{11} & \mathbf{N}_{12} & \mathbf{R}^T \\ \mathbf{N}_{21} & \mathbf{N}_{22} + \alpha\mathbf{I} & \mathbf{0} \\ \mathbf{R} & \mathbf{0} & \mathbf{0} \end{pmatrix} \begin{pmatrix} \hat{\mathbf{x}} \\ \hat{\mathbf{z}} \\ \hat{\mathbf{k}} \end{pmatrix} = \begin{pmatrix} \mathbf{U}_1 \\ \mathbf{U}_2 \\ \mathbf{w} \end{pmatrix} \tag{6}$$

with $\mathbf{N}_{11}=\mathbf{A}^T\mathbf{P}\mathbf{A}$, $\mathbf{N}_{12}=\mathbf{A}^T\mathbf{P}\mathbf{B}$, $\mathbf{N}_{21}=\mathbf{B}^T\mathbf{P}\mathbf{A}$, $\mathbf{N}_{22}=\mathbf{B}^T\mathbf{P}\mathbf{B}$, $\mathbf{U}_1=\mathbf{A}^T\mathbf{P}\mathbf{y}$ and $\mathbf{U}_2=\mathbf{B}^T\mathbf{P}\mathbf{y}$. $\mathbf{I}$ is ($t_2\times t_2$) identity matrix. We also define the following notations

$$\begin{cases} \mathbf{N}_\alpha = \mathbf{N}_{22} - \mathbf{N}_{21}\tilde{\mathbf{Q}}_{11}\mathbf{N}_{12} + \alpha\mathbf{I} = \tilde{\mathbf{N}}_{22} + \alpha\mathbf{I} \\ \tilde{\mathbf{Q}}_{11} = \mathbf{N}_{11}^{-1} - \mathbf{N}_{11}^{-1}\mathbf{R}^T\mathbf{N}_R\mathbf{R}\mathbf{N}_{11}^{-1} \\ \mathbf{N}_R = \left(\mathbf{R}\mathbf{N}_{11}^{-1}\mathbf{R}^T\right)^{-1} \\ \tilde{\mathbf{U}}_2 = \mathbf{U}_2 - \mathbf{N}_{21}\tilde{\mathbf{Q}}_{11}\mathbf{U}_1 \\ \mathbf{u} = \mathbf{N}_{11}^{-1}\mathbf{R}^T\mathbf{N}_R\mathbf{w} \end{cases} \tag{7}$$

The regularized float ambiguities with constraints are

$$\hat{\mathbf{z}}_{\alpha,f} = \mathbf{N}_\alpha^{-1}\left(\tilde{\mathbf{U}}_2 - \mathbf{N}_{21}\mathbf{u}\right) \tag{8}$$

and the other real-valued parameters

$$\begin{aligned}\hat{\mathbf{x}}_{\alpha,f} &= \tilde{\mathbf{Q}}_{11}\mathbf{U}_1 - \tilde{\mathbf{Q}}_{11}\mathbf{N}_{12}\mathbf{z}_{\alpha,f} + \mathbf{u} \\ &= \tilde{\mathbf{Q}}_{11}\mathbf{U}_1 - \tilde{\mathbf{Q}}_{11}\mathbf{N}_{12}\mathbf{N}_\alpha^{-1}\tilde{\mathbf{U}}_2 + \left(\tilde{\mathbf{Q}}_{11}\mathbf{N}_{12}\mathbf{N}_\alpha^{-1}\mathbf{N}_{21} + \mathbf{I}\right)\mathbf{u}\end{aligned} \tag{9}$$

In line with the law of covariance propagation, the covariance matrix of the regularized float ambiguities is derived as

$$\boldsymbol{\Sigma}_{\hat{\mathbf{z}},f} = \sigma_0^2\mathbf{N}_\alpha^{-1}\left(\mathbf{N}_{22} - 2\mathbf{N}_{21}\tilde{\mathbf{Q}}_{11}\mathbf{N}_{12} + \mathbf{N}_{21}\tilde{\mathbf{Q}}_{11}\mathbf{N}_{11}\tilde{\mathbf{Q}}_{11}\mathbf{N}_{12}\right)\mathbf{N}_\alpha^{-1} \tag{10}$$

It is important that the float solution is biased due to the regularization criterion. Its bias is computed as

$$\mathbf{b}_\alpha = -\alpha\mathbf{N}_\alpha^{-1}\bar{\mathbf{z}} \tag{11}$$

where $\bar{\mathbf{z}}$ is the "true" ambiguity vector and is replaced by its LS solution with constraints in practice. The mean squares error (MSE) is used to describe the accuracy of regularized solution considering the effect of the bias term

$$\mathbf{M}_{\hat{\mathbf{z}},f} = \mathbf{\Sigma}_{\hat{\mathbf{z}},f} + \mathbf{b}\mathbf{b}^T = \sigma_0^2 \mathbf{N}_\alpha^{-1}\Big(\mathbf{N}_{22} - 2\mathbf{N}_{21}\tilde{\mathbf{Q}}_{11}\mathbf{N}_{12} + \mathbf{N}_{21}\tilde{\mathbf{Q}}_{11}\mathbf{N}_{11}\tilde{\mathbf{Q}}_{11}\mathbf{N}_{12}\Big)\mathbf{N}_\alpha^{-1} + \alpha^2\mathbf{N}_\alpha^{-1}\bar{\mathbf{z}}\bar{\mathbf{z}}^T\mathbf{N}_\alpha^{-1} \tag{12}$$

There are many methods to estimate the regularization parameter, and we numerically compute it based on minimizing the trace of $\mathbf{M}_{\hat{\mathbf{z}},f}$. For more information, one can refer to Xu (1998). Once the regularization parameter is properly determined, equation (8) can give the float ambiguities that are normally closer to the true integer ambiguities than LS solutions, and their MSE matrix computed from (12) would be less correlated than LS covariance matrix. Similar to the integer LS AR, we establish the following criterion for integer searching which is considered more efficient relative to the LS,

$$\Omega = \left(\hat{\mathbf{z}} - \mathbf{z}_{\alpha,f}\right)^T \mathbf{M}_{\hat{\mathbf{z}},f}^{-1}\left(\mathbf{z} - \mathbf{z}_{\alpha,f}\right) = \min \tag{13}$$

where $\hat{\mathbf{z}}$ is the vector of integer candidates to be searched. Once the ambiguity is correctly fixed, the real-valued parameters are updated as

$$\hat{\mathbf{x}} = \mathbf{x}_{\alpha,f} - \tilde{\mathbf{Q}}_{11}\mathbf{N}_{12}\left(\mathbf{z} - \mathbf{z}_{\alpha,f}\right) \tag{14}$$

2.2 Three reduced models from the generalized model

Up to now, we have derived the generalized AR model based on the regularization criterion extended with available constraints. We will discuss its three reduced models by specifying the variables in Eqs.(7) to (14).

- If the LS is used instead of regularization to solve the float solution and the constraints are still available,

  $$\alpha = 0,\ \mathbf{N}_\alpha = \tilde{\mathbf{N}}_{22},\ \mathbf{b}_\alpha = \mathbf{0},\ \mathbf{M}_{\hat{z},f} = \mathbf{\Sigma}_{\mathbf{z},f} \tag{15}$$

  Li and Shen (2009) have specified several constraints that can enhance the float solutions.
- If the regularization criterion is still used but without available constraints

  $$\mathbf{R} = \mathbf{0},\ \ \mathbf{w}=\mathbf{0},\ \ \mathbf{N}_R = \mathbf{0},\ \ \mathbf{u}=\mathbf{0},\ \ \tilde{\mathbf{Q}}_{11} = \mathbf{N}_{11}^{-1} \tag{16}$$

  This model has been carefully addressed by Shen and Li (2007), and results showed that the AR can indeed improved in the case of rapid GPS positioning.
- If the LS is used without any available constraints,

  $$\begin{cases}\alpha = 0,\ \mathbf{R} = \mathbf{0},\ \ \mathbf{N}_R = \mathbf{0},\ \ \mathbf{w}=\mathbf{0},\ \ \mathbf{u}=\mathbf{0},\ \ \tilde{\mathbf{Q}}_{11} = \mathbf{N}_{11}^{-1} \\ \mathbf{N}_\alpha = \mathbf{N}_{22} - \mathbf{N}_{21}\mathbf{N}_{11}^{-1}\mathbf{N}_{12},\ \tilde{\mathbf{U}}_2 = \mathbf{U}_2 - \mathbf{N}_{21}\mathbf{N}_{11}^{-1}\mathbf{U}_1\end{cases} \tag{17}$$

  It is the standard integer LS model for AR.

3 Methods of ambiguity searching

Once the integer searching criterion (13) is established, the next procedure is to efficiently determine the optimal candidate, because there is a very huge number of candidates, especially in the case of highly correlated matrix $\mathbf{M}_{\hat{\mathbf{z}},f}$ and large number of unknown ambiguities, need to test by substituting them into (13). Actually, most of AR methods have been developed to improve the searching efficiency.

3.1 Exclusion of nuisance ambiguity candidates

In the searching process, the constraints on the integer ambiguities should be employed. We firstly check all integer candidates using the integer constraints (3). If the integer candidates are not compatible with the constraints (3), they are immediately excluded. In addition, we can construct some statistics for ambiguities themselves or for their linear combinations to exclude some nuisance candidates thus exempting them from further testing. The essence of the "FARA" method proposed by Frei and Beutler (1990) is to construct the statistics for all ambiguities and for the difference of any two ambiguities as integer constraints to efficiently exclude most of ambiguity candidates.

In the short-baseline kinematic GPS positioning, there are three unknown coordinates besides integer ambiguities. In fact three ambiguity-fixed DD observations can be used to determine the coordinates and the others are totally redundant. In other words, once three ambiguities are fixed, the coordinates can be solved and then the other ambiguities can be trivially fixed, which is the essence of the LS ambiguity searching technique and ARCE (ambiguity resolution using constraint equation) proposed by Hatch (1990) and Park et al. (1996), respectively. In fact, both of them use the relationship among ambiguities of single epoch where DD phase equations are of rank defect with the number of three (Li and Shen 2010). In long-baseline ambiguity resolution with three or multiple frequency signals, the extra-widelane and widelane ambiguities can be firstly reliably fixed and then used as integer constraints for narrow-lane ambiguity resolution (Feng 2008; Li et al. 2010).

In general, a great number of examples have proven that these integer constraints mentioned above are always very strong and can be used to eliminate most of nuisance integer candidates (Li and Shen 2010). After this operation, if there are still more than one integer candidate, we will compute their corresponding quadratic values Ω by (13) and further choose the optimal integer candidate with employment of other indicators.

3.2 Rounding: the simplest method

The earliest and easiest method of AR is rounding off to fix the float ambiguity directly to its nearest integer if both its fraction and uncertainty are small enough,

$$\hat{z}_i = \mathrm{round}\left(z_{i,f}\right) \tag{18}$$

where $\hat{z}_i$ and $\hat{z}_{i,f}$ are the ith fixed ambiguity and float one, respectively. In principle, the rounding is only applied to consider the diagonal elements of the covariance matrix of float ambiguities, and their correlations are totally disregarded. Therefore, it usually takes a long time such that the float ambiguity has sufficiently small fraction and variance to warrant a correct integer solution.

3.3 Bootstrapping: partially considering the correlation

Another relatively simple integer estimator method is the bootstrapping estimator, which is initially applied by Dong and Bock (1989) and elaborated by Teunissen (1999). The implementation of bootstrapping is similar to rounding except the correlation between float ambiguities is taken into account. It is essentially a sequentially conditional adjustment and the new float ambiguities can be relatively more precisely fixed by use of their correlation with the fixed ambiguities. In theory, it is still a rounding method and the float solution transformed by the *cholesky decomposition* is used. Usually, it starts from the ambiguity with smallest variance. However, the overall correlations of all ambiguities are not fully considered after all. Thus, the success rate of the bootstrapping solution is lower than the rigorous Integer Least-Squares solution as discussed below.

3.4 Integer LS: the rigorous method

In order to obtain the optimal integer solution, the rigorous covariance matrix of the float solution must be used. The problem is that the original covariance matrix is highly correlated so that the determined searching intervals of all ambiguities are always large for including the true integer candidate. In other words, there are a huge number of potential candidates in the super-ellipsoid governed by this covariance matrix. Fortunately, this problem was solved through the decorrelation technique firstly introduced by Teunissen (1993). The decorrelation technique decreases the correlation of covariance matrix of the float solution and then reduces the number of integer candidates. As a result, the AR efficiency is significantly improved and the optimal solution is achievable. Afterwards, the decorrelation technique has been intensively investigated and three general techniques are proposed, namely, LLL (Grafarend 2000), integer cholesky decomposition and inverse integer cholesky decomposition (Xu 2001), of which the integer cholesky decomposition has been essentially applied in the LAMBDA method.

3.5 Optimal combinations for TCAR

One of the benefits from three and multiple frequency GNSS signals offered by future systems is to form more useful combinations that generally have smaller total noise level in cycles or reduced ionosphere effects (Feng 2008). As a result, the combined float ambiguities can be determined more precisely and easily fixed to their integer values. For instance, the ambiguities of two extra-widelane combinations can be determined with both the geometry-based and geometry-free models at success rate of 99% and above almost instantaneously without distance constraint (Feng 2008, Li et al. 2010). However the narrow-lane AR over long baselines is still problematic mainly due to the effect of distance-dependent residual tropospheric biases after the effect of the ionospheric biases are reduced in the optimal combinations. The statistic results from the semi-generated three frequency GPS signals as shown in Li (2008), have shown that the success probabilities of narrow-lane instantaneous AR solutions are about 83% and 24% for 53km and 155km baselines, respectively (Li et al. 2010). In the network-based AR, the AR allows for use of accumulated measurements from multiple epochs. Li et al. (2010) proposed a both geometry-free and ionosphere-free model that overcomes the geometric and ionospheric effects simultaneously without constraints on inter-station distances, and 100% AR success rate can be achieved using measurements of several minutes.

4 Experiment and analysis

We demonstrate the superior performance of regularized AR using the real single-frequency GPS data. There are total 8000 epochs with sampling interval of 1s and baseline length of 4.6km. The elevation mask angle is set to 13 degrees. 10 epochs of phase data are used to compute the regularized and LS float ambiguities at each performance. The differences between the solved float ambiguities and their true integer values are shown in Figure 1. The differences are tens of cycles for LS and reduced to smaller than 3 cycles for regularization, which means that the float solution can be significantly improved.

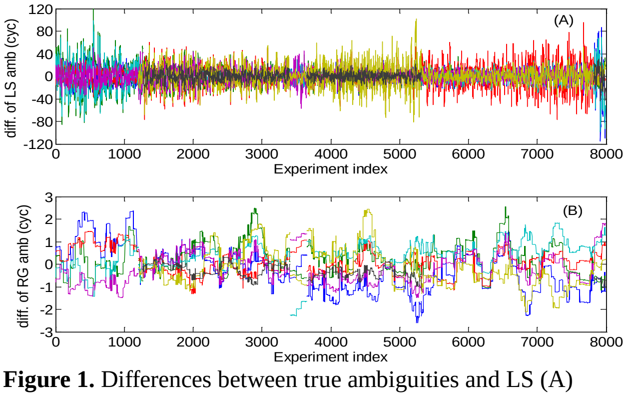

Figure 1. Differences between true ambiguities and LS (A) ambiguities as well as regularized ambiguities (B)

Table 1: Success AR probabilities for regularization (RG) and LS estimations with different number of epochs

Method	Number of epochs			
	3	5	10	30
LS	0.11	1.03	2.44	10.34
RG	59.51	59.44	60.13	62.77

We further assess the success probability of regularized AR with different numbers of epoch data. The success probability is defined to be a ratio of the number of epochs with all correctly fixed ambiguities over the number of total epochs. The results are given in Table 1. The success probability can be significantly improved for all computation schemes at different degree. In general, when more data epochs are involved, the observation geometry becomes stronger and thus the power of regularization becomes smaller.

We turn to analyse the benefit of the constraint to the float solution using another set of real dual-frequency GPS data with the baseline length of about 10 km. A total of 600 epochs were collected with sampling interval of 1 s and data from 6 common satellites (i.e., total 10 DD ambiguities). For both LS and regularization processes we use 10 epoch phase data to compute the LS float solutions with or without different constraints, i.e., baseline length constraint, horizontal coordinate constraint as well as 3D coordinate constraint, respectively. There are total 591 computations and the differences between float ambiguities and their true integer values are shown in Figure 2. Apparently, the float solution can be significantly improved by using the constraint, and the stronger is the constraint, the better is the float solution. Especially for the case of 3D coordinate constraint, the float ambiguities are sufficiently precise such that they can be immediately fixed by rounding off to their nearest integers.

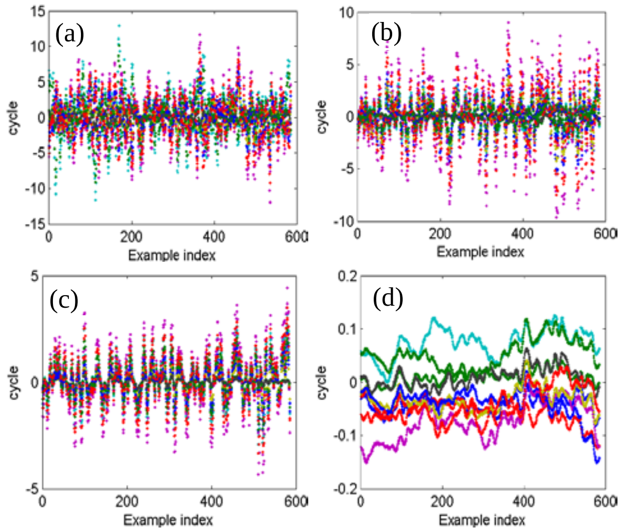

Figure 2. Differences between float ambiguities and their true values without constraint (a) and with baseline length constraint (b), with horizontal coordinate constraint (c) and 3D coordinate constraint (d), respectively

5 Concluding remarks

In this paper, we have presented an extended model for efficient AR with constraints. In this model, the regularization criterion is used instead of the traditional LS criterion such that the ill-condition problem of the LS normal equations of AR can be significantly mitigated. As a result, better float ambiguity solutions can be derived and the covariance matrices are less correlated. From this extended model, one can deduce three reduced models widely refereed in the existing literatures. In addition, we have introduced the exclusion of nuisance integer candidates based on the available constraints as the important first step of the integer search. Finally, the numerical experiments are carried out to demonstrate the superior performance of regularized AR and the benefit of constraints to the float solution. In general, the regularization can mitigate the ill-condition of fast GNSS AR model and thus improve the success AR probability. The constraint can improve the float solution and especially for the case of strong constraints the float ambiguities can be improved to such precision that they can be directly fixed by rounding off to their nearest integers.

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