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Time Estimation of Superimposed Coherent Multipath Signals Using the EM Algorithm for Global Positioning System

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Abstract. A novel multipath mitigation technique for Global Positioning System (GPS) receivers using the Expectation-Maximization (EM) algorithm is proposed. It is well-known that conventional propagation delay estimation using parallel sliding correlators is only optimal in additive white Gaussian noise channel. In practical positioning systems, the weak GPS line-of-sight signal is generally embedded in the multipath signals and other source of interference. Although the GPS direct sequence spread spectrum (DS-SS) signal has inherent resistance to interference, the received superimposed multipath signals, which are possibly coherent, are the dominant source of the propagation delay estimation errors. From the parameter estimation point of view, the problem of multipath mitigation is equivalent to estimating the unknown phases, propagation delays and amplitudes of the superimposed multipath signals. The joint maximum likelihood (ML) estimation of all the unknown parameters is optimal and asymptotically efficient. However it involves multi-dimensional search which is computationally expensive. The proposed coarse/acquisition (C/A) code acquisition system using the EM algorithm is an iterative maximum likelihood estimator which decomposes the multi-parameter estimation problem into a number of separate ML optimizations. The performance of the proposed EM algorithm has been tested by simulations. We have observed that the proposed acquisition system is significantly superior to the conventional correlating receiver in a multipath fading channel.

Keywords: Multipath Mitigation, Expectation-Maximization (EM) Algorithm, Time Estimation, Code Acquisition

1 Introduction

The focus of this paper is on multipath mitigation of the received superimposed signals in Global Positioning System (GPS). The operating principle of radio navigation systems is based on the propagation time estimation of the broadcast signals from the constellation satellites. In practical wireless channel, the transmitted signals are propagated along various reflected paths to the receive antenna. These replicas of the received signals are known as the multipath signals. For Global Navigation Satellite Systems, the prime information in concern is the exact propagation time of the direct signal from the satellite to the receiver. The reflected signals arrived at the antenna not only convey no information about for the pseudorange measurement, but they also induce errors for the geometric distance determination. Although the GPS direct sequence spread spectrum (DS-SS) signal has inherent resistance to interference, however, the received superimposed multipath signals, which are possibly coherent, are the dominant source of the propagation delay estimation errors.

In this paper, the problem of multipath cancellation is tackled in terms of the parameter estimation point of view. Specifically, by estimating all the unknown phases, amplitudes and the time delays of the reflected signals, the effect of the undesirable multipath signals can be minimized. It is well-known from the estimation theory that, the maximum likelihood (ML) estimators are optimal and asymptotically efficient. However, the cost function of the ML estimator is a nonlinear function of the unknown time delays of the reflected signals. In addition, the joint estimation of all the unknown parameters is a multi-dimensional minimization problem which is computationally expensive.

In the literature, the Multipath Estimating Delay Lock Loop (MEDLL) (van Nee 1992; van Nee *et al.* 1994) is a maximum likelihood estimator which is tailored to a multipath propagation environment. The weak power of the received GPS signals is highly susceptible to jamming signals and unintended interferences. The MEDLL is similar to a conventional correlator, except that it is optimized with respect to a multipath fading channel rather than a Gaussian noise channel. By estimating the amplitudes, delays and phases of all the D identified multipath signals simultaneously, the MEDLL can significantly reduce the measurement errors induced by the multipaths. The iterative ML estimator proposed in this paper is similar to the MEDLL in the sense that, it also has the multipath suppression capabilities.

A novel GPS receiver endowed with the multipath mitigation capabilities is proposed in this paper. Essentially, the proposed acquisition system decomposes the multi-parameter estimation problem into a number of separate ML optimizations, and hence, the computational cost is reduced. The propagation time of the line-of-sight signal and other unknown parameters are estimated in an iterative manner. The code acquisition system adopted the well-known Expectation-Maximization (EM) algorithm (Dempster *et al*., 1977), which is an iterative maximum likelihood estimator. In Section 2, the theoretical outline of the EM algorithm is described. The proposed code acquisition architecture for the GPS using the EM algorithm is presented in Section 3. The simulation results are presented in Section 4. It is shown that the performance of the proposed receiver architecture is significantly improved in a multipath fading environment.

2 The Expectation-Maximization (EM) algorithm

The problem of GPS C/A code acquisition in a multipath fading environment using the Expectation-Maximization (EM) Algorithm is considered in this paper. The EM algorithm is an iterative computation routine of maximum likelihood estimates. The EM algorithm is closely related to estimation with missing data; it is particularly applicable to incomplete data problems. In the literature, the term complete data **X** generally refers to the parameter bearing data which lies on the sample space Ω. On the other hand, the term incomplete data **Y** refers to the sampled vector that lies on the observation space Φ. The complete data **X** depends on a set of unknown parameter θ, which lies on a measurable parameter space, denoted as Θ. Suppose that there exists a many-to-one transformation $\mathrm{T} : \Omega \rightarrow \Phi$. The parameter embedded data **X** is not directly observed, but instead, it is observed through the transformation T.

2.1 Outline of the EM algorithm

Let $L(Y;\theta)$ and $L_c(X;\theta)$ be the likelihood function of the observed (incomplete) data and the complete data, respectively. The maximum likelihood estimate $\hat{\theta}_{ML}$ is the parameter which maximizes $L_c(X;\theta)$, i.e.

$$\hat{\theta}_{ML} = \arg\max_{\theta} L_c(X;\theta) \quad (1)$$

However, the complete data **X** is not observable under the transformation T and its probability density function $p(X;\theta)$ is unknown, and hence the ML estimate $\hat{\theta}_{ML}$ in (1) cannot be evaluated directly. The approach taken by the EM algorithm is to evaluate the expected likelihood function $L_c(X;\theta)$ instead. Specifically, the estimate is obtained by maximizing the expected value of the complete data log likelihood function, given the observation and a preliminary estimate of the unknown parameter, which is denoted as θ'. Mathematically, the EM estimate $\hat{\theta}_{EM}$ is given by

$$\hat{\theta}_{EM} = \arg\max_{\theta} E\left[\log L_c(X;\theta) \mid Y, \theta'\right] \quad (2)$$

Due to the fact that the EM estimate $\hat{\theta}_{EM}$ is dependent on the preliminary estimate θ', hence $\hat{\theta}_{EM}$ can be considered as an improved estimate over θ'. Subsequently, the updated estimate can be used as a preliminary estimate and equation (2) can be evaluated repeatedly with an improved estimate at each iteration. The EM algorithm is therefore an iterated estimator, in the sense that, given an initial guess θ_0, the estimate $\hat{\theta}_{EM}$ can be obtained by evaluating (2) iteratively. Essentially, the EM algorithm involves two main steps,

for k = 1,2,…

E-Step

$$U(\theta;\hat{\theta}^{(k)}) = E\left[\log L_c(X;\theta) \mid Y, \hat{\theta}^{(k)}\right] \quad (3)$$

M-Step

$$\hat{\theta}^{(k+1)} = \arg\max_{\theta} U(\theta;\hat{\theta}^{(k)}) \quad (4)$$

Under normal conditions, the estimate $\hat{\theta}_{EM}$ will eventually converge to the maximizer of the complete data log likelihood function in equation (1). Although the convergence to the global maximizer is not guaranteed for multimodal cost functions, but if the initial guess is sufficiently close to the global maximum, in most cases, convergence to the global maximum can be achieved. The issue of convergence of the EM Algorithm will be discussed in the next section.

2.2 Convergence of the EM algorithm

In this section we will outline the proof which shows that the EM Algorithm estimate $\hat{\theta}_{EM}$ would converge to the maximum likelihood estimate $\hat{\theta}_{ML}$ as the number of iterations increases. For a more detailed discussion, the readers are recommended to refer to the seminal paper by Dempster et al. (Dempster *et al.,* 1977). The conditional density probability function is given by $p(X \mid Y;\theta) = p(X \mid \theta)/p(Y \mid \theta)$, the log likelihood of the incomplete data can be written as

$$\log L(Y;\theta) = \log L_c(X;\theta) - \log p(X \mid Y;\theta) \quad (5)$$

By taking the conditional expectation of equation (5) given the observation Y with a preliminary estimate of the unknown parameter θ', we have

$$\begin{aligned}&\log L(Y;\theta)\\ &= E\left[\log L_c(X;\theta) \mid Y,\theta'\right] - E\left[\log p(X \mid Y;\theta) \mid Y,\theta'\right]\\ &= U(\theta;\theta') - H(\theta;\theta')\end{aligned}$$

where

$$H(\theta;\theta') = E\left[\log p(X \mid Y;\theta) \mid Y,\theta'\right]$$
$$U(\theta;\theta') = E\left[\log L_c(X;\theta) \mid Y,\theta'\right]$$

By performing the E-step and M-step iteratively, we obtain a sequence of estimates $\{\hat{\theta}_k\}$ (k=1,2,…). The difference of the log likelihood $L(Y;\theta)$ between two successive estimate can be written as

$$\begin{aligned}&\log L(Y;\theta^{(k+1)}) - \log L(Y;\theta^{(k)})\\ &= \left\{U(\theta^{(k+1)};\theta^{(k)}) - U(\theta^{(k)};\theta^{(k)})\right\} -\\ &\quad \left\{H(\theta^{(k+1)};\theta^{(k)}) - H(\theta^{(k)};\theta^{(k)})\right\}\end{aligned} \quad (6)$$

The first bracket in (6) is the difference of conditional expectation of the complete data. It is chosen to be greater or equal to zero, this is essentially the M-Step of the EM algorithm as stated in (4). The second bracket in (6), on the other hand, is the difference of the conditional expectation of the conditional probability density. It can be shown by using the Jensen's inequality that the difference is less than or equal to zero. For any estimate θ',

$$\begin{aligned}&H(\theta';\theta^{(k)}) - H(\theta^{(k)};\theta^{(k)})\\ &= E\left[\log\left\{p(X \mid Y;\theta')/p(X \mid Y;\theta^{(k)})\right\} \mid Y,\theta^{(k)}\right]\\ &\le \log\left[E\left\{p(X \mid Y;\theta')/p(X \mid Y;\theta^{(k)})\right\} \mid Y,\theta^{(k)}\right]\\ &= \log\int p(X \mid Y,\theta)dx\\ &= 0\end{aligned}$$

The above integral is over the complete data **X** which lies on the range of the transformation mapping T . By combining the above results, we have shown that $\log L(Y;\theta^{(k+1)}) \ge \log L(Y;\theta^{(k)})$, hence the estimation sequence $\{\hat{\theta}_k\}$ progressively increases the log likelihood function. In other words, the EM estimate is approaching to the maximum likelihood estimate as the number of iteration increases.

3 GPS signal acquisition using the EM algorithm

The weak received GPS signal is vulnerable to interfering signals. Generally, the received signal is a superimposed of the distorted multipath signals, intentional or unintentional interference and channel noise. In particular, the close-in coherent multipath reflections of the direct signal are difficult to distinguish from the desired signal. It can cause serious contamination of the observable measurements. Conventional correlator-based detectors provide no protection from multipath and interference; these disturbances are collectively regarded as Gaussian channel noise in general. In a wireless multipath environment, the navigation accuracy can be degraded significantly if the detector fails to combat against the multipath reflections and interferences. There have been efforts devoted to improve the jamming immunity and apply special multipath rejection techniques at the GPS receivers, e.g. MEDLL (van Nee 1992; van Nee *et al.* 1994). The proposed C/A code acquisition system using the EM Algorithm will be described in this section.

3.1 System model

The standard GPS C/A code has a chip rate of $1/Tc$ = 1.023MHz, one period of the spreading code consists of P = 1023 chips, hence it spans in 1 millisecond. Suppose that a total number of D superimposed signals are detected at the receive antenna. Let T be the symbol transmission time, the C/A spreading waveform of the k^{th} received superimposed signal during the l^{th} signaling interval $[(l\text{-}1)T, lT]$ can be mathematically represented as

$$c_k(l) = \sum_{i=0}^{P-1} q_k(i)p(l - iT_c),$$

where $p(\cdot)$ is the chip waveform with duration T_c and $\{q_k(i)\}_{i=0}^{P-1} \in \{-1,1\}^P$ represents the chip sequence of the pseudo-random (PRN) code. The message transmission rate in GPS is $1/T$ = 50Hz, so that the spreading factor is T/T_c = 20460. Suppose that the received waveform is sampled at a rate Q/T_c, where Q is an integer which represents the oversampling factor.

Let τ_i be the unknown propagation time delay of the i^{th} multipath signal. Without loss of generality, we assume $\tau_0 \le \tau_1 \le ... \le \tau_{D-1}$ so that τ_0 represents the propagation time of the line-of-sight signal. Since the propagation delays of the transmitted waveforms are unknown, the received sequence is not synchronized. In other words, during the m^{th} sampling bit interval at the receiver, it may span across the boundary of two transmitted data bits. In order to account for this fact, let us define $q_k^{(j)}$ and s_k to be the augmented zero-padding sampled vectors of the C/A code waveform of the k^{th} superimposed signal. Both $q_k^{(j)}$ and s_k are $2PQ\times 1$ real vectors, specifically they are defined as follows

$$q_k^{(j)} = [\underbrace{0,...,0}_{j}, \underbrace{q_k(0),...,q_k(0)}_{Q},...,$$
$$\underbrace{q_k(P-1),...,q_k(P-1)}_{Q}, \underbrace{0,...,0}_{PQ-j}]^t,$$
$$s_k = (1 - \delta_k Q/T_c)q_k^{(i_k)} + (\delta_k Q/T_c)q_k^{(i_k+1)},$$

where

$$i_k = \left\lfloor \frac{Q\tau_k}{T_c} \right\rfloor, \quad \text{and} \quad \delta_k = \tau_k - \frac{i_k T_c}{Q}, \qquad k = 0,...,D-1$$

The two scalars i_k and δ_k account for the delay of the sampled signal. Let $d_k(m) \in \{-1,1\}$ be the data bit of the k^{th} superimposed signal during the m^{th} symbol interval. Let

$y(m) = [y(mPQ + PQ),..., y(mPQ + 1)]^t \in C^{PQ}$ and
$w(m) = [w(mPQ + PQ),..., w(mPQ + 1)]^t \in C^{PQ}$,

be the discrete received signal vector and the noise vector during the m^{th} symbol interval, respectively. The $PQ\times 1$ sampled received vector $y(m)$ during the m^{th} symbol interval can be represented as

$$y(m) = \sum_{k=0}^{D-1} \alpha_k (d_k(m-1)s_k^l + d_k(m)s_k^u) + w(m), \qquad (7)$$

where $s_k^l = [0_{PQ} \quad I_{PQ}]s_k$, $s_k^u = [I_{PQ} \quad 0_{PQ}]s_k$. $\alpha_k \in C$ is the unknown complex channel coefficient of the k^{th} superimposed signal detected at the sensor. If the m^{th} symbol interval spans across two message data bits, the vector s_k^l corresponds to the chip sequence of the first data bit, i.e. $d_k(m-1)$, and s_k^u corresponds to the chip sequence of the second data bit $d_k(m)$. The graphical illustration of the received C/A code sequence is depicted in Figure 1. The black box which spans for a time T, which corresponds to the received C/A code sequence with the same data bit. The C/A code sequence inside the dotted box corresponds to the received coded sequence during the m^{th} symbol interval at the receiver. The channel is assumed to be slowly varying in the sense that the channel coefficient, α_k, is an unknown deterministic constant during the observation time. The noise vector $w(m)$ is assumed to be Gaussian distributed with covariance matrix Σ_w.

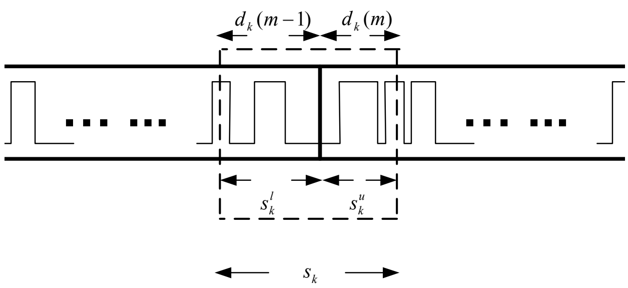

Fig. 1 Sampled received C/A code of the kth multipath signal during the mth symbol interval

3.2 Time delay estimation of superimposed multipath signals

We assumed that the Doppler shift is known perfectly, so that the problem of signal acquisition is equivalent to estimating the code shift. The system model of the superimposed GPS spreading signals is shown in (7). In order to mitigate multipath signals, the time delays $\tau = [\tau_0, \tau_1, ..., \tau_{D-1}]^t$ and the channel coefficients

$\alpha = [\alpha_0,...,\alpha_{D-1}]^t$ for all D superimposed signals are required to be estimated.

The ML estimate of the time delay τ is a nonlinear multi-dimensional optimization problem. In order to formulate the EM algorithm, the equation in (7) is rewritten as follows

$$\begin{aligned} y(m) &= \sum_{k=0}^{D-1} \alpha_k (d_k(m-1)s_k^l + d_k(m)s_k^u) + w(m) \\ &= \sum_{k=0}^{D-1} \alpha_k u_k(m;\tau) + w(m) \\ &= \sum_{k=0}^{D-1} (\alpha_k u_k(m;\tau) + w_k(m)) \\ &= T \cdot x(m;\theta), \end{aligned}$$

where

$$u_k(m;\tau) = d_k(m-1)s_k^l + d_k(m)s_k^u,$$

$$w(m) = \sum_{k=0}^{D-1} w_k(m),$$

$$x_k(m;\theta) = \alpha_k u_k(m;\theta) + w_k(m), \tag{8}$$

$$x(m;\theta) = [x_0(m;\theta), x_1(m;\theta),..., x_{D-1}(m;\theta)]^t,$$

$$T = \left[\underbrace{I_{PQ}...I_{PQ}}_{D} \right].$$

Recall that Σ_w is the covariance matrix of the noise vector $w(m)$. We assume that the noise vectors $w_k(m)$, $k = 1,2,..., D-1$ are statistically independent, zero-mean and Gaussian distributed with covariance matrix $E(w_k(m)w_k^H(n)) = \Sigma^{(k)}\delta(m-n)$, such that $\Sigma^{(k)} = \beta_k \Sigma_w$. We denote $\delta(\cdot)$ as the Kronecker delta function. The parameters β_k can be adjusted under the constraint

$$\sum_{k=0}^{D-1} \beta_k = 1.$$

The matrix **T** is a many-to-one transformation matrix. The parameter bearing data vector $x(\theta)$ is referred as the complete data and the observation vector $y(m)$ is considered as the incomplete data. The navigation data message $d_k(m-1)$ and $d_k(m)$ are unknown to the receiver prior to code acquisition. In this paper, two assumptions have been made for the formulation of the EM algorithm.

Time delay estimation in CDMA multiple access channel has been widely investigated in the mobile communication research community, e.g. Strom *et al.,* 1996. The data bits for each superimposed signal is generally modelled as independently distributed, so that the propagation delay can be estimated by using subspace methods, e.g. MUSIC (Schmidt, 1986). However, the superimposed GPS multipath signals are possibly coherent, i.e. identical spreading code and data bits. It is particularly true for the close-in scattered signals which are reflected in the vicinity of the receiver. These close-in multipath signals which arrive only slightly later than the line-of-sight signal, usually also possess comparable power as the line-of-sight signal. These are particularly harmful for positioning systems. Due to the coherence of the reflected signals, the data bits of the detected multipath signals are generally identical, i.e. $d_0(m) = d_1(m) = ... = d_{D-1}(m)$. Hence the subspace methods are not applicable in this case, and ML estimation is adopted instead.

Secondly, one period of C/A code spans a time of 1ms, the dwell time used for code searching using the conventional correlators is therefore a multiple of 1ms. A long dwell time can be chosen to improve the acquisition of the weak satellite signals. A high probability of detection and small probability of false alarm can also be achieved simultaneously by using a longer dwell time. However, the GPS navigation message is transmitted at a rate of 50Hz, hence there is a sign reversal at most once every 20ms. This sets the limit of the data length for acquisition. In practice, the dwell time ranges from 1ms to 4ms depending on the SNR of the received signal. Since the close-in multipath signals arrive only slightly later that the desired line-of-sight signal, we assume that there is no sign reversal occurs for all received multipath signals during the dwell time.

3.3 Successive multipath suppression using the EM algorithm

Let us define $u(m;\tau) = [u_0(m;\tau), u_1(m;\tau),..., u_{D-1}(m;\tau)]^t$ and $\alpha = [\alpha_0,...,\alpha_{D-1}]^t$. By using (8), the log-likelihood of the complete data $x(m;\theta)$ is given by

$$\log f_x(x;\theta) = C - (x(m;\theta) - \alpha^H u(m;\tau))^H \Lambda^{-1} (x(m;\theta) - \alpha^H u(m;\tau)) \tag{9}$$

where $\Lambda = Diag(\Sigma^{(0)},...,\Sigma^{(D-1)})$ is a block diagonal matrix and C is a constant which is independent on the parameter θ. However, the complete data $x(\theta)$ is unobserved; the actual distribution cannot be analytically derived. With an initial estimate of the unknown parameters $\hat{\theta}^{(0)} = [\hat{\alpha}^{(0)}, \hat{\tau}^{(0)}]$, the EM estimate iterates until the current estimate $\hat{\theta}^{(k)} = [\hat{\alpha}^{(k)}, \hat{\tau}^{(k)}]$ is sufficiently close to the previous estimate $\hat{\theta}^{(k-1)}$, i.e.

$\left\|\hat{\theta}^{(k)}-\hat{\theta}^{(k-1)}\right\|<\varepsilon$, for an arbitrary small value ε. Let us denote the conditional expectation of the complete data as

$$U(\theta;\hat{\theta}^{(k)})=E\left\{\log f_x(x;\theta)\mid y,\hat{\theta}^{(k)}\right\} \tag{10}$$

From (8), it is clear that the received vector $y(m)$ and the complete data $x(m;\theta)$ are jointly Gaussian distributed. The complete and the incomplete data are related by the transformation **T**. In particular, the parameter bearing data $x(m;\theta)$ has a mean $\alpha^H u(m;\tau)$ with covariance matrix $\Sigma^{(k)}=\beta_k\Sigma_w$.

The E-Step of the EM algorithm requires taking the conditional expectation of equation (9) stated above. Hence, it is necessity to evaluate $\hat{x}(m;\theta)=E[x(m;\theta)\mid y(m)]$. From the classical estimation theory, the conditional expectation $\hat{x}(m;\theta)$ is given by

$$\hat{x}(m;\theta)=E[x(m;\theta)]+\Sigma_{xy}\Sigma_y^{-1}(y(m)-E[y(m)]) \tag{11}$$

where Σ_{xy} is the cross covariance matrix of $x(m;\theta)$ and $y(m)$, Σ_y is the covariance matrix of $y(m)$. Due to the diagonal structure of the covariance matrix Λ, and by using (11), the conditional expectation of the j^{th} multipath data signal can be written as

$$\begin{aligned}\hat{x}_j(m;\hat{\theta}^{(k)})&=\hat{\alpha}_j^{(k)}u_j(m;\hat{\tau}^{(k)})+\beta_j(y(m)-\\&\sum_{n=0}^{D-1}\hat{\alpha}_n^{(k)}u_n(m;\hat{\tau}^{(k)})),\quad j=0,1,...,D-1\end{aligned} \tag{12}$$

The E-Step of the EM algorithm involves taking the conditional expectation $U(\theta;\hat{\theta}^{(k)})$ in (10), it can be expressed as

$$\begin{aligned}U(\theta;\hat{\theta}^{(k)})=C-(\hat{x}(m;\hat{\theta}^{(k)})-\alpha^H u(m;\tau))^H\Lambda^{-1}\\(\hat{x}(m;\hat{\theta}^{(k)})-\alpha^H u(m;\tau))\end{aligned} \tag{13}$$

The conditional expectation $\hat{x}(m;\theta)$ is given in equation (12). On the other hand, the M-Step of the EM algorithm is to locate the parameter $\hat{\theta}$ which maximizes (13), i.e.

$$\hat{\theta}^{(k+1)}=[\hat{\alpha}^{(k+1)},\hat{\tau}^{(k+1)}]=\arg\max_{\theta}U(\theta;\hat{\theta}^{(k)})$$

Recall that $\Lambda=Diag(\Sigma^{(0)},...,\Sigma^{(D-1)})$, where $\Sigma^{(k)}=\beta_k\Sigma_w$. By using (12) and (13), the j^{th} multipath complex channel coefficient and its EM time delay estimation during the k^{th} iteration is given by

$$\hat{\tau}_j^{(k+1)}=\arg\max_{\tau}\left|\hat{x}_j(m;\hat{\theta}^{(k)})^H u_j(m;\tau)\right| \tag{14}$$

and

$$\hat{\alpha}_j^{(k+1)}=\frac{\hat{x}_j(m;\hat{\theta}^{(k)})^H u_j(m;\hat{\tau}^{(k+1)})}{u_j(m;\hat{\tau}^{(k+1)})^H u_j(m;\hat{\tau}^{(k+1)})} \tag{15}$$

We note from (14) that the time delay of the j^{th} multipath signal $\hat{\tau}_j^{(k+1)}$ is obtained by correlating the estimated multipath signal $\hat{x}_j(m;\hat{\theta}^{(k)})$ with the C/A spreading code $u_j(m;\tau)$. This can be done by using the conventional non-coherent combining method (Kaplan, 1996) or the subspace method (Schmidt, 1986). The channel coefficient estimate in (15) mimics to the Linear Minimum Mean Square Error (LMMSE) estimate. Note also that the denominator of (15) is the energy of the spreading code $u_j(m;\hat{\tau}^{(k+1)})$. It can be considered as a constant. The channel coefficient $\hat{\alpha}_j^{(k+1)}$ is evaluated by using the updated time delay $\hat{\tau}^{(k+1)}$ obtained in (14).

To summarize, the multipath time delay estimation using the EM algorithm involves the following steps:

Initialize $\hat{\theta}_0$ with an initial estimate.

For k = 1,2,…

E-Step

$$\begin{aligned}\hat{x}_j(m;\hat{\theta}^{(k)})&=\hat{\alpha}_j^{(k)}u_j(m;\hat{\tau}^{(k)})+\beta_j(y(m)-\\&\sum_{n=0}^{D-1}\hat{\alpha}_n^{(k)}u_n(m;\hat{\tau}^{(k)})),\quad j=0,1,...,D-1\end{aligned}$$

M-Step

$$\hat{\tau}_j^{(k+1)}=\arg\max_{\tau}\left|\hat{x}_j(m;\hat{\theta}^{(k)})^H u_j(m;\tau)\right|$$

$$\hat{\alpha}_j^{(k+1)}=\frac{\hat{x}_j(m;\hat{\theta}^{(k)})^H u_j(m;\hat{\tau}^{(k+1)})}{u_j(m;\hat{\tau}^{(k+1)})^H u_j(m;\hat{\tau}^{(k+1)})}$$

A block diagram of the time delay EM estimate is given in Figure 2. The block "Correlator" in the figure corresponds to the non-coherent correlation in (14). The block "LMMSE" refers to the LMMSE channel coefficient estimation as given in (15). The block "SMS" corresponds to the estimated multipath signals decomposition in equation (12). The resultant signal $\hat{x}_j(m;\hat{\theta}^{(k)})$ is the j^{th} multipath signal estimated at the k^{th} iteration.

Remark 1: The initial estimate can be chosen randomly, for instance, in our simulation, the initial time delay $\hat{\tau}_0$ is randomly chosen in the range [0ms 1ms]. The initial channel coefficient $\hat{\alpha}$ can be chosen arbitrary, the choice of $\hat{\alpha}$ depends highly on the actual SNR of the channel. However, a wise choice of $\hat{\alpha}$ significantly improves the convergence rate of the algorithm.

Remark 2: There are a total number of D distinct multipath time delays $\hat{\tau}_j$ ($j = 0,1..., D-1$) to be estimated. The cost function in (14) is multimodal, convergence to the global maximum is not guaranteed. In some cases, we observed that the time delay estimate of

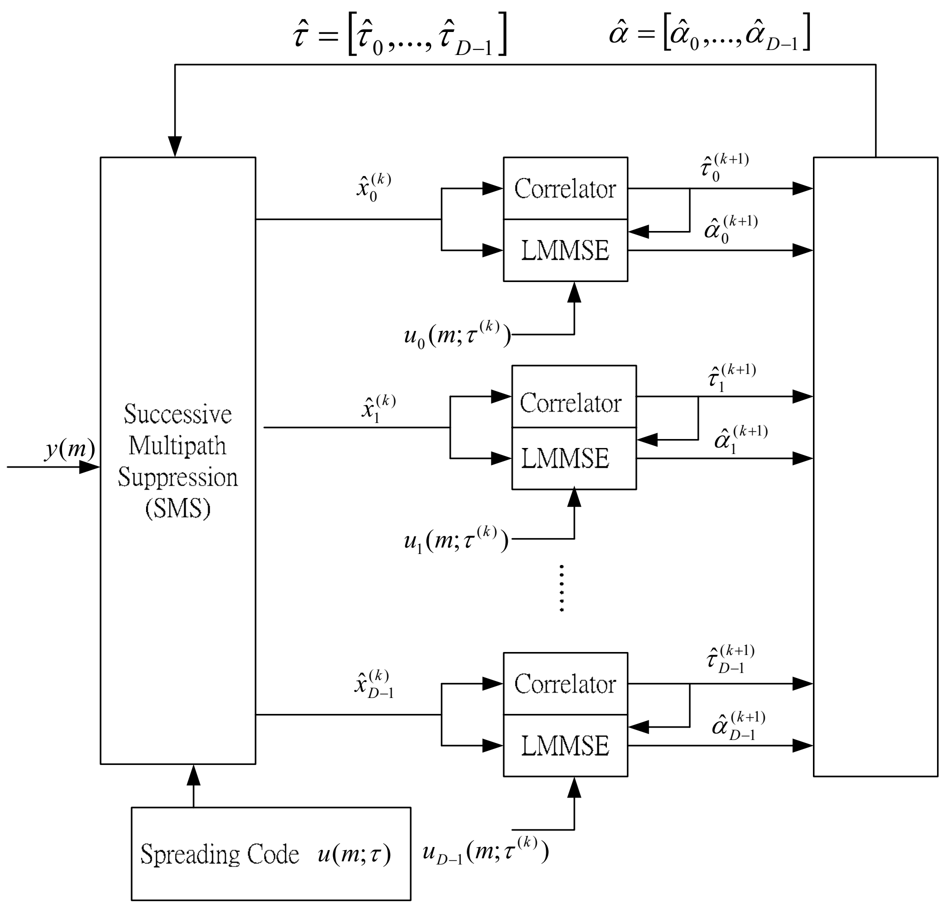

Fig. 2 Block diagram of the successive multipath suppression using the EM algorithm

two or more multipath signals give the same values (i.e. $\hat{\theta}_n^{(k)} = \hat{\theta}_m^{(k)}$, $m \neq n$), so that less than D multipath delay estimates are given by the algorithm. This is due to the fact that the resultant estimate falls into the same local stationary value. However, this can be alleviated by re-initializing the intermediate estimate $\hat{\theta}_j^{(k)}$ to an arbitrary value, so that it can follow another iterative path to converge to the desired time delay.

Remark 3: We can see from (12) that $\hat{x}_j(m;\hat{\theta}^{(k)})$ is simply the estimated j^{th} multipath signal obtained by subtracting all other contributing multipath signals estimated at the previous iteration.

4 Simulation results and discussions

In order to demonstrate and evaluate the proposed iterative signal acquisition algorithm, a number of simulations have been performed. Suppose that two close-in multipath signals along with the line-of-sight signal superimposed signals are detected at the receiver. The SNR of the received line-of-sight signal is -23dB, while the ratio of the line-of-sight signal to each multipath signal is 3dB. For each experiment, the observation time and the acquisition dwell time of the correlator is 2ms, the signal is oversampled by a factor of five (i.e. $Q = 5$), so that a total number of 10000 samples are received at the GPS receiver during the observation time.

For the sake of illustrating the performance of the proposed algorithm, one of the experimental data is shown in Table 1. The actual time delays for the direct signal and each multipath signal are $\tau_0 = 0.3412ms$, $\tau_1 = 0.5344ms$ and $\tau_2 = 0.7272ms$, respectively. The real channel coefficients have values $\alpha_0 = 0.1001$ for the line-of-sight signal, and $\alpha_1 = \alpha_2 = 0.0709$ for the two multipath signals. The initial estimate of the unknown parameters, α and τ are chosen randomly as:

$\alpha_0 = [0.0136,\ 0.0104,\ 0.0099]^t$

$\tau_0 = [0.4112,\ 0.3093,\ 0.8398]^t$

Five iterations are performed and non-coherent combining is performed in parallel for each superimposed signal. It shows that both the estimation errors for the

Tab. 1 Experiment data of time delay estimates of two multipath signals with a direct signal.

Iteration	$\hat{\tau}_0$	$\hat{\tau}_1$	$\hat{\tau}_2$	$\hat{\alpha}_0$	$\hat{\alpha}_1$	$\hat{\alpha}_2$	$\lVert \tau - \hat{\tau} \rVert^2$	$\lVert \alpha - \hat{\alpha} \rVert^2$
1	**0.341200**	**0.341200**	**0.341200**	**0.013602**	**0.010420**	**0.009897**	**0.18630**	**0.01486**
2	**0.341200**	**0.534400**	**0.534400**	**0.048723**	**0.010420**	**0.009897**	**0.03720**	**0.01002**
3	**0.341200**	**0.534400**	**0.698800**	**0.072219**	**0.027579**	**0.009897**	**0.00081**	**0.00638**
4	**0.341200**	**0.534400**	**0.698800**	**0.087977**	**0.045961**	**0.016689**	**0.00081**	**0.00371**
5	**0.341200**	**0.534400**	**0.698800**	**0.098534**	**0.058260**	**0.027845**	**0.00081**	**0.00202**

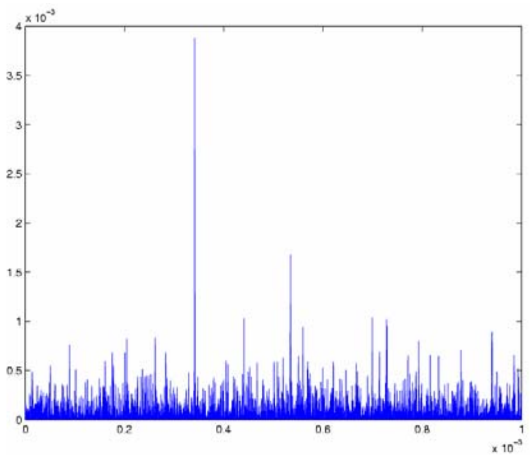

Fig. 3 Non-coherent combining with conventional sliding correlator

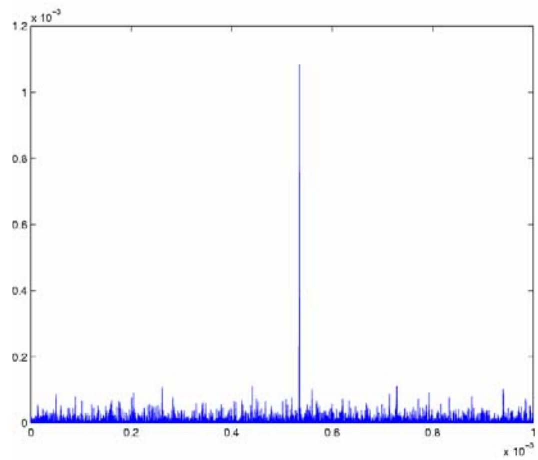

Fig. 5 Non-coherent combining using EM algorithm. The peak corresponds to the time delay estimate of the first multipath signal

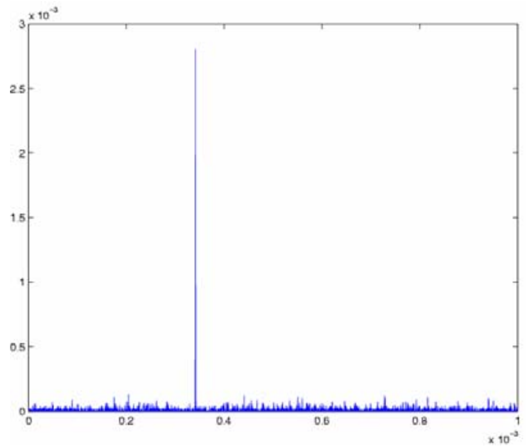

Fig. 4 Non-coherent combining using EM algorithm. The peak corresponds to the time delay estimate of the direct signal

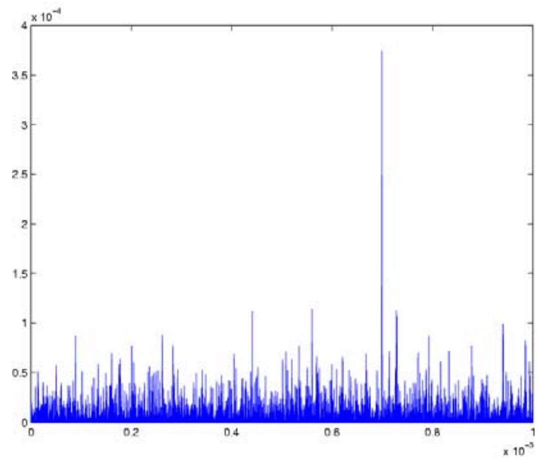

Fig. 6 Non-coherent combining using EM algorithm. The peak corresponds to the time delay estimate of the second multipath signal

time delay estimate and the channel coefficient estimate gradually decreases with the number of iterations performed.

Note that during the first and second iterations, two or more delay estimates give the same values (e.g. $\hat{\tau}_1^{(2)} = \hat{\tau}_2^{(2)} = 0.5344$), it is due to the fact the estimates converge to the same stationary points. This can be remedied by re-initializing the trapped values (e.g. $\hat{\tau}_2^{(2)}$ is set to a random value in the range [0ms 1ms]) at the next iteration, so that the next estimate relies on a new initial estimate, rather than the previous stationary estimate $\hat{\tau}_2^{(2)}$. This has been discussed in Remark 2 of the previous Section.

The correlator output with the conventional sliding correlator is shown in Figure 3. The peaks correspond to the time delays of the two multipath signals and the line-of-sight signal. The correlator output of the direct signal (i.e. $\hat{x}_0(\theta)$ in Figure 2) using the proposed signal acquisition technique with the EM algorithm is shown in Figure 4, we can see that the multipath signals have been suppressed substantially. Hence the probability of false alarm can be significantly reduced and the probability of detection can be increased simultaneously. Figure 5 and Figure 6 show the correlator outputs for the other two multipath signals. Our simulation results show that the proposed GPS signal acquisition system using the EM algorithm provide a multipath suppression capability which is highly attractive in multipath propagation environment.

5 Conclusions

A novel GPS C/A code acquisition system tailored to the multipath propagation environment is proposed in this paper. The proposed GPS receiver jointly estimates the time delays of all the multipath signals simultaneously by using the well-known EM algorithm. The time delays are estimated in an iterative manner, and the multipath signals can be subsequently suppressed. The iterative maximum likelihood estimator is asymptotically efficient. In particular, the pseudorange accuracy can be improved substantially in the presence of coherent multipath signals.

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