

# Real Time Quality Assessment for CORS Networks

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Received: 15 November 2004 / Accepted: 13 July 2005

**Abstract.** The growing use of real time high accuracy Global Positioning System (GPS) techniques has resulted in an increase in the number of critical decisions made on the basis of a GPS derived position. When making these decisions mobile users require assurance that the GPS position quality meets their requirements. Providers of Continually Operating Reference Stations (CORS), whom mobile users are generally reliant upon, must also be able to assure users that their data meets agreed quality standards. Unfortunately, the realistic and reliable description of position and data quality is an area in which GPS has traditionally been weak. Research being undertaken as part of the Cooperative Research Centre for Spatial Information (CRC-SI) is attempting to address this problem by assessing and reporting on the quality of raw GPS observations in real time. This paper examines a number of existing approaches to assessing the quality of raw GPS observations and presents a conceptual architecture for the development of a real time quality control system.

**Key words:** GPS, Quality, Real Time, Stochastic Modelling, CORS

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## 1 Introduction

The increasing usage of high accuracy real time GPS positioning in a wide range of applications has resulted in a proportional rise in the number of critical decisions made on the basis of GPS positions. These decisions may be critical from a safety-of-life, financial, or environmental perspective. In making these decisions GPS users must be capable of determining if the quality of the position meets their requirements. Furthermore, they must be confident that the indicators of position quality that their decision is based on are realistic and reliable in all conditions, at any time.

To obtain high accuracy real time GPS positions, mobile users rely heavily on information from external sources, be it from a local GPS basestation, a regional CORS network, or a global correction service. Thus the quality of the mobile user's position is intrinsically linked to the quality of the external data. Mobile users must be assured that the information provided to them is of sufficient quality to meet their requirements. It follows that suppliers of GPS data products, e.g. CORS providers, must be able to deliver quality information to the mobile user in real time.

Research being undertaken as part of the positioning program of the CRC-SI is attempting to address many of the issues associated with the real time assessment of CORS and mobile user positioning quality. The aim of this research is to develop real time procedures for CORS networks and mobile users that will improve the reliability of the mobile user's position and provide a realistic assessment of the position quality. To accomplish this an understanding of existing approaches to quality control and the ability of these approaches to be adapted to real time operation is required. This paper presents a review of the current methods for assessing the quality of CORS and mobile user data and positions, in conjunction with an analysis of the potential of these methods to operate in real time. Finally a conceptual architecture for the real time quality control of CORS networks and mobile users is proposed.

## 2 Quality control for CORS networks and mobile users

The positioning accuracy and quality achievable by GPS is dependent on the raw data quality and the processing algorithm chosen. The quality control of GPS observations falls into two parallel categories – the validation and description of the raw data quality, independent of its future application, and the quality

control undertaken as part of the processing algorithm (Brown et al., 2003). Given the wide range of processing algorithms available the quality control processes employed by these algorithms is not of particular concern at this stage. Suffice it to say that the quality control and end results of the chosen processing algorithm will be dependent on the provision of the high quality observation data and an accurate stochastic model, both of which are a direct outcome of quality control of raw observation data.

The methods and procedures for the validation and description of raw data quality are generally independent of the processing algorithm chosen. The aspects of raw data quality control considered here include data completeness, the detection and repair of cycle slips and receiver clock jumps, and the description of raw data quality in the form of stochastic models.

### 2.1 Data completeness

The most basic form of raw data quality control consists of statistics that describe the amount and completeness of the data collected by a GPS receiver. The consequences of ignoring data completeness in a quality control process can be severe, leading to difficulty in detecting outliers and cycle slips, increased time to resolve ambiguities, the introduction of multiple ambiguities, a weaker solution due to limited data availability, and in the worst case an inability to compute a solution (Brown et al., 2003). Three aspects of data completeness are generally considered; data gaps - being epochs with incomplete or no observations; missing epochs - whereby observations are not recorded for a satellite that is visible; and the availability of sufficient ephemeris information for a satellite. Statistics on data completeness, when analysed over extended time periods, can be useful for determining problems with receiver hardware and software (Brown et al., 2003), site-specific problems (Brown et al., 2003, Jonkman and de Jong, 2000a), and abnormalities in the satellite constellation or ephemeris information (Jonkman and de Jong, 2000a). Current quality control software packages such as GQC (Brown et al., 2003) and TEQC (Estey and Meertens, 1999) operate in a post-processing mode and are well suited to this sort of task.

From a real time quality control (RT-QC) perspective data gaps, missing epochs, satellite constellation problems and so forth need to be closely monitored and appropriate action taken to notify users of any problems that may impact on the quality of their position solution. Additionally, data gaps and missing epochs are likely to have a detrimental impact on the ability of any real time algorithms for the detection of cycle slips, or the generation of stochastic models, to carry out their assigned tasks..

### 2.2 Systematic biases in observation data

High accuracy GPS positioning is dependent upon the identification and removal of the main error sources that impact upon the observation quality. In relation to the quality control of raw data, receiver clock jumps, cycle slips, and quasi-random (e.g. multipath, diffraction, ionospheric scintillation etc.) effects are the main error sources that can degrade observation quality. The impact of quasi-random errors are not considered here but are dealt with briefly in the section describing stochastic modelling. However, the influence of quasi-random errors does hamper cycle slip detection, mainly due to the fact that their influence on the phase observations is not limited to an integer number of cycles (Kim and Langley, 2001). The treatment of true systematic errors in the RT-QC context is discussed in the following sections.

#### 2.2.1 Receiver clock jumps

GPS receivers align themselves with GPS time using a variety of techniques. Some receivers constantly synchronise their clock with GPS time (so called "Clock Steering") whilst others allow their clock to drift and periodically introduce corrections of approximately 1 millisecond to keep the clock close to GPS time (Fig. 1). Other receivers allow the clock to drift unchecked and simply keep track of the bias and bias rate of change (Rizos, 1999, Gurtner, 1999, Fraser, 2004).

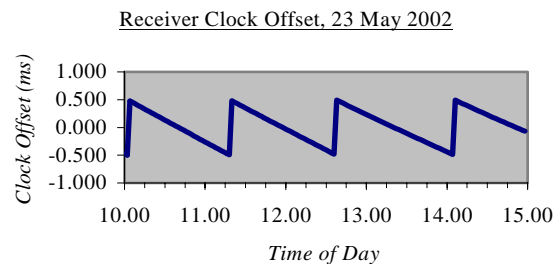


Fig 1: Receiver Clock Jumps

Of concern from a RT-QC perspective is the second technique (illustrated in Fig. 1), whereby clock jumps are introduced into the raw observations. These jumps produce a systematic bias in the undifferenced code and phase observations, as shown in the following equation:

$$\Phi(t + \Delta) = \Phi(t) + \dot{\Phi} \cdot \Delta = \Phi(t) + \dot{\rho} \cdot \Delta - c \cdot \Delta \quad (1)$$

where  $\Phi$  represents the carrier phase (or pseudorange) observation and  $\dot{\Phi}$  its rate of change with respect to time;  $\Delta$  represents the clock jump;  $\dot{\rho}$  is the satellite dependent geometric range rate; and  $c$  is the speed of light in a vacuum. The clock jumps themselves are quite

small (less than or equal to 1 millisecond) but they have two distinct effects on the code and phase observables. The term  $c \cdot \Delta$  represents a constant receiver dependent effect on the geometric range whilst the term  $\dot{\rho} \cdot \Delta$  represents the contribution of the satellite dependent geometric range rate at the time of the clock jump (Kim and Langley, 2001). The first of these terms ( $c \cdot \Delta$ ) is removed during subsequent single or double difference processing. The latter term ( $\dot{\rho} \cdot \Delta$ ) does not cancel during differencing, as it is dependent on a particular satellite-receiver combination.

Thus the term  $\dot{\rho} \cdot \Delta$  introduces a systematic bias into the geometric range rate. The size of the bias is dependent on the particular geometric range rate. Assuming a maximum possible rate of 900m/s, a one-millisecond jump could potentially introduce 0.9m of error into the geometric range. From a RT-QC perspective it is crucial that these effects are estimated and removed in real time. Without correcting for such an effect it may be difficult to detect and repair cycle slips, estimate an accurate stochastic model, and undertake subsequent quality control (e.g. during the processing algorithm).

### 2.2.2 Cycle slip detection and repair

Cycle slips are discontinuities of an integer number of cycles in the carrier phase observations caused by a loss of lock in the receiver's carrier tracking loops. Hofmann-Wellenhof et al. (1992) describe three potential causes for cycle slips. Firstly, the most likely cause of cycle slips are physical obstructions to the satellite signal due to natural or man-made features (e.g. buildings, trees, bridges etc.). Secondly, low signal to noise ratios (SNR) due to ionospheric conditions, multipath, rapid changes in receiver position, or low satellite elevation can produce cycle slips. Finally, failures in the receiver software or malfunctioning satellite oscillators may cause cycle slips, however such incidents are rare.

To take advantage of the superior measurement precision of the phase observables, cycle slips must be removed from the phase data before further processing can occur. This process involves detecting the location of the cycle slip (in time), determining the number of L1 and/or L2 cycles that comprise the slip, and then correcting all phase observations of the affected satellite subsequent to the time of the cycle slip (Kim and Langley, 2001, Hofmann-Wellenhof et al., 1992).

The focus on Real Time Kinematic (RTK) positioning in recent times has moved the detection and repair of cycle slips, traditionally a post-processed activity, into the real-time domain. RTK positioning is dependent on the resolution of the integer ambiguities, a process greatly aided by the presence of clean, cycle slip free data. The

push for instantaneous ambiguity resolution has led to the development of real-time algorithms for the detection and repair of cycle slips.

One such algorithm is the instantaneous cycle slip correction technique proposed by Kim and Langley (2001). This algorithm utilises the triple difference (TD) observables of the carrier phases in conjunction with Doppler and code observables. TD observations are generally free of the majority of GPS biases, such as receiver and satellite clock offsets, integer ambiguities, atmospheric effects, multipath, and satellite orbits. Thus, the size of the remaining biases and noise should be less than a few centimetres, provided that the observation interval is relatively short. Cycle slips would then be evident in the TD observations as large spikes, several orders of magnitude larger than the mean bias and noise. These assumptions may not hold in all cases, for example severe ionospheric disturbances, very long baselines, or rapid variations in the receiver position may lead to the triple difference biases and noise exceeding the L1 and L2 wavelengths, without cycle slips being present. In such situations the observation interval can be reduced to a level such that the biases and noise exhibited by the TDs are once again at the centimetre level and therefore, useful in detecting and repairing cycle slips (Kim and Langley, 2001).

Cycle slip candidates are obtained by examining the mean and variance of the predicted TD residuals (being the difference between the observed TDs and the computed TDs). If dual frequency carrier phase observations are available the number of candidates can be reduced through the use of TDs formed from the geometry free linear combination observations. Following identification of the cycle slip candidates a least squares estimation is carried out to determine the two candidates (best and second best) that minimise the least squares residuals. The statistical likelihood of these two candidates is assessed and if they are considered significantly different then the best candidate is accepted and the slip is repaired. In a final step a reliability test on the cleaned data is carried out to determine if further, unspecified, errors remain in the observations.

Another example of an algorithm capable of real-time cycle slip detection and repair has been proposed by de Jong (1998) and was implemented in the Dutch Permanent GPS Network and during the International GLONASS Experiment (Jonkman and de Jong, 2000b). The algorithm is based on the use of a Kalman filter in conjunction with the recursive Detection, Identification and Adaptation (DIA) procedure developed by Teunissen (1990). The DIA procedure consists of an overall model test to detect any unspecified errors in the observation or dynamic models (Detection). If an unspecified error is encountered a number of alternative models, incorporating different bias parameters, are tested. The

model producing the highest test statistic is considered the most likely to represent the “correct” observation model (Identification). Finally the original observation model is modified to reflect the identified bias (Adaptation).

The DIA algorithm was developed to be independent of the positioning application the data was intended for. Thus no external information such as receiver-satellite geometry, clock offsets or atmospheric should be required. This is accomplished through the use of the geometry free linear combination for both the observation and dynamic models (Jonkman and de Jong, 2000b). Of particular note is that this method is applied on a satellite-by-satellite basis for a single receiver, thus no observation differencing is required. This is advantageous in the sense that data from other receivers is not required to detect cycle slips. Further studies by de Jong (1998) showed that the DIA geometry free approach, on a satellite-by-satellite basis, was theoretically capable of detecting slips of a single cycle in magnitude, provided the observation interval is relatively short.

### 2.3 Stochastic Modelling

GPS data processing involves the determination of various unknown parameters (e.g. station coordinates, tropospheric estimates, integer ambiguities etc.) from a set of observations. Generally these observations consist of different types of measurements on different frequencies (e.g. code and phase measurements on L1 and L2) and there are usually large numbers of them when compared to the unknown parameters. In the positioning community the accepted methodology for determining the parameters is least squares (LSQ) estimation. LSQ estimation relates the observed quantities to the unknown parameters through a set of mathematical equations known as a functional model. The noise or precision of the observed quantities is represented using a stochastic model.

A great deal of work has been put into the development of functional models for GPS data processing. The stochastic model has received less attention from researchers until relatively recently. As a result simple stochastic models are frequently used in LSQ based GPS data processing algorithms (Tiberius et al., 1999).

Stochastic models are used in three phases of GPS processing, quality control of the raw observations, ambiguity resolution, and computation of the unknown parameters (Kim and Langley, 2001). The statistical quantities used in cycle slip detection and repair algorithms are derived from the chosen stochastic model. Thus the use of incorrect or oversimplified stochastic models may result in faulty slip detection, thereby introducing biases into the ambiguity resolution and

parameter estimation processes. Similarly, the performance of instantaneous, real-time ambiguity resolution strategies is greatly improved when using accurate stochastic models. Accurate stochastic models reduce the ambiguity search space and ensure that the fixed ambiguities are correct. An incorrect stochastic model could potentially result in faulty ambiguity resolution, with unsatisfactory consequences for the accuracy of the positioning application. Finally, the estimated quality of the unknown parameters (obtained from the LSQ estimation) are implicitly dependent on the a priori stochastic model. An incorrect a priori model may lead to overly optimistic estimates of the derived position quality, leading users to believe they have met quality requirements when, in fact they have not (Tiberius et al., 1999).

A number of methods have been proposed to provide more realistic stochastic models for the various GPS observables. Four approaches will be considered here - the elevation dependent method (Euler and Goad, 1991), the SNR or  $C/N_0$  approach (Brunner et al., 1999, Richter and Euler, 2001), a rigorous least squares estimation approach, and a method based on time differencing (Kim and Langley, 2001).

#### 2.3.1 Elevation dependent modelling

The dependence of observation noise on satellite elevation has been known for some time and can mainly be attributed to the receiver antenna’s gain pattern, with additional contributions from atmospheric attenuation and multipath (Kim and Langley, 2001, Tiberius et al., 1999). Modelling the observation noise with respect to satellite elevation can be carried out using functions tailored to individual receivers (Euler and Goad, 1991) or using general functions that can be applied regardless of receiver type (Hugentobler et al., 2004). One drawback of the elevation dependent approach is that it only considers the variance of the individual observations. Cross correlations between observations types (e.g. C1 and P2) are neglected, as are spatial and temporal correlations. Thus a fully populated variance covariance matrix is not available when using this method.

#### 2.3.2 $C/N_0$ Based modelling

GPS signal power is expressed in the form of carrier-to-noise power density ratios ( $C/N_0$ ), also known as signal to noise ratios (SNR). The  $C/N_0$  measurements generated by GPS receivers are an indication of how well the receiver hardware is tracking the incoming GPS signals. As such they provide a direct indication of the quality of the phase observations (Richter and Euler, 2001, Kim and Langley, 2001, Brunner et al., 1999). The  $C/N_0$  approach to

stochastic modelling seeks to take advantage of this information to provide a more realistic assessment of the observation noise.

$C/N_0$  values are highly correlated with satellite elevation, due in the most part to the antenna gain pattern, but also influenced by atmospheric refraction and multipath. Initial work focussed on this link to produce stochastic models that were in effect, elevation dependent (Hartinger and Brunner, 1999). Further work by (Brunner et al., 1999) extended the simple  $C/N_0$  models to account for the fact that  $C/N_0$  is also influenced by signal diffraction.  $C/N_0$  values observed in “clean” environments can be treated as a “known” template for  $C/N_0$  values observed in other environments. Deviations of the observed values from the template are considered to be the result of diffraction and down weighting (or removal) of the observations occurs as a result. The practical difficulties of providing templates for the various receiver-antenna combinations has been discussed in Richter and Euler (2001).

Problems with this method include the dependence on  $C/N_0$  values, which may not be available from all receivers, and the fact that cross, spatial, and temporal correlations are not considered.

### 2.3.3 Least squares estimation

The least squares estimation approach offers a rigorous solution to the problem of estimating a priori stochastic models. Results in Barnes et al. (1998) indicate that using the optimal stochastic model, estimated from the LSQ residuals, significantly effects positioning results, when compared to alternative modelling approaches (e.g.  $C/N_0$  approach). The basis of this approach is the direct estimation of every element in the a priori variance covariance matrix from the a posteriori observation residuals. Due to the recursive nature of this process it can be incorporated into a Kalman filter or sequential least squares adjustment (Kim and Langley, 2001).

One technique to carry out the estimation of the variance covariance elements is Minimum Norm Quadratic Unbiased Estimation (MINQUE) developed by Rao (1971) and utilised for static baseline processing by Wang (1998). Unfortunately, MINQUE and similar techniques are computationally intensive and not suited to real-time processing. The optimality of the least squares estimation approach is not guaranteed, as the estimation technique may make assumptions about the correlations that do not hold in all cases (e.g. temporal correlations

may be ignored). Furthermore, a certain level of observation redundancy is required to produce reasonable estimates, a situation that may not exist in all positioning scenarios (Kim and Langley, 2001).

### 2.3.4 Differencing in the time domain

Differencing in the time domain was proposed by Kim and Langley (2001) to overcome the three main problems in the existing modelling approaches - the lack of a fully populated variance covariance matrix, no temporal correlations, and no observation redundancy in long baseline solutions. This method takes the view that high order differencing in time (differencing TDs to produce quadruple differences (QDs), then differencing QDs to produce quintuple differences (dQDs)) will remove all systematic biases and correlations, leaving only white noise.

The assumption that systematic biases and correlations are removed is justified on the basis that the differencing process is in effect the application of consecutive subtractive filters. These filters remove biases (e.g. receiver and satellite clock offsets), damp low frequency effects (e.g. atmospheric, multipath), and amplify high frequency effects (e.g. noise, ionospheric scintillation). For short baselines the effects of the correlated biases are assumed to be ignorable, thereby implying the temporal correlations are also ignorable. However, temporal correlations may still exist, particularly in high multipath environments, thus high order differencing is still required. For long baselines the correlated biases are not ignorable and consequently time correlations will exist (Kim and Langley, 2001). Assuming the dQDs are free of systematic biases and correlations they represent white noise at the dQD level. The variance covariance matrix of the dQDs can then be formed from a set of arbitrary dQD samples. Using the mathematical relationship between the various differencing levels, variance covariance matrices for any difference (i.e. zero, single, double) can be derived.

Of concern here is the generation of the dQD variance covariance matrix. One solution is the estimation of covariance functions. However, this is a computationally intensive process not really suited to real-time use. If a simpler technique is utilised one must question its effectiveness in correctly modelling the cross, spatial, and temporal correlations, particularly when extrapolating back from the dQDs. Furthermore, this method is

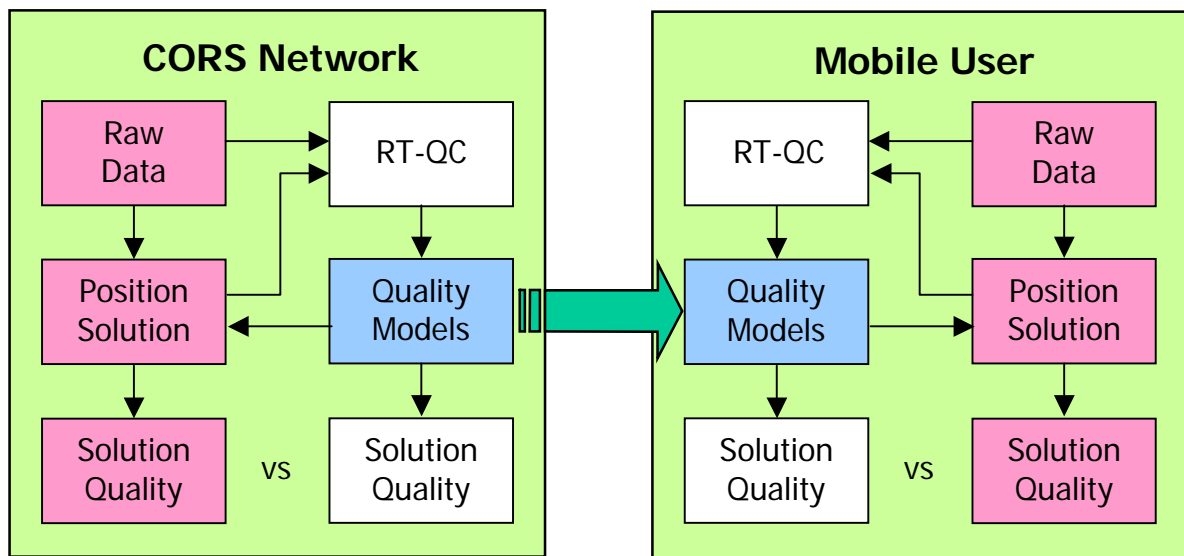


Fig. 2: Proposed RT-QC Architecture.

dependent on the selection of an appropriate time interval for the differencing. The assumption that the dQD observable represents white noise requires the high frequency biases and correlations (which are amplified by the use of subtractive filters) to be insignificant. This may not always be the case (e.g. in unstable ionospheric conditions) and it may be necessary to adjust the time interval in response to changes in the behaviour of the high frequency biases.

### 3 RT-QC Architecture

The aim of the research being undertaken is the development of real time procedures for CORS networks and mobile users that will improve the reliability of the mobile user's position and provide a realistic assessment of the position quality. Through an examination of the existing approaches to assessing raw data quality an understanding of the various aspects and limitations of raw data quality assessment has been developed. To proceed further, a conceptual architecture for a proposed RT-QC system has been developed and is shown in Fig 2.

The RT-QC architecture is built around the idea that the assessment of raw data quality (RT-QC box) should be carried out independent of the processing algorithm (Position Solution box). However, in the initial stages of the project information from the position solution will be considered during the quality control process. The red boxes indicate the current approach to assessing the quality of CORS and mobile users position and raw data. As Fig. 2 shows, this research is attempting to develop procedures whereby quality models of the CORS network data can be transmitted to a mobile user, thereby

improving the quality of the mobile user's position and the estimates of position quality.

### 4 Conclusions

The number of critical decisions made on the basis of GPS positions has increased proportionally with the use of GPS within the community. When faced with a decision that may have severe consequences GPS users must be confident that their position has been determined to a sufficient level of quality to justify the decision and that the indicators of quality their decisions is based are realistic and reliable. The quality of a GPS position is a direct result of the raw data quality and the processing algorithm chosen. This paper has presented a review of some existing methods for the assessment and reporting of raw GPS data quality and the potential of these methods to be adapted for use in a real time environment. A conceptual architecture of a RT-QC system has been presented as a way forward for future research in this area.

### Acknowledgements

This work has been undertaken as part of Project 1.2 in the CRC-SI. The authors would like to acknowledge the support of the industry partners in this project.

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