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Triple-Frequency Method for High-Order Ionospheric Refractive Error Modelling in GPS Modernization

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Abstract. New opportunities for the refinement of ionospheric modelling and reduction of the ionospheric error in GPS measurements arise since a third-frequency will be introduced for the modernised GPS system. This paper investigates theoretical models of the ionospheric refractive error. A triple-frequency method of correcting the 1st and 2nd order ionospheric refraction is presented and a triple-frequency ionosphere-free combination method is proposed for GPS modernization. These new methods can be equally applied to the European GALILEO system. In addition, typical effects of the 2nd and 3rd order ionospheric effects are also investigated, and a correct formula for the 3rd order ionospheric error is derived in a simple way for easy implementation.

It is anticipated that the proposed refraction correction methods will play an important role in both the modernized GPS and GALILEO systems. Results show that the proposed triple-frequency methods can correct the ionospheric refraction effects to a millimetre level. Since the models are given in simple forms, these corrections can be easily implemented in many real-time applications and the triple-frequency methods for correcting high-order ionospheric can significantly eliminate the error remained in the current ionospheric models. The method developed will potentially contribute to a better long-range baseline ambiguity resolution and an accuracy improvement in precise point positioning.

Key words: GPS Modernization, Ionospheric Refractive Error, Triple-Frequency Methods, GALILEO.

1 Introduction

It is well known that ionospheric refractive error is one of the main factors that restrict the GPS positioning accuracy. The equivalent distance error of the ionospheric effect is typically about 15 meters in daytime, 3 meters in the evening, and a maximum of 50 meters in the zenith and 150 meters in the horizon (Zhou, 1995). Therefore, the ionospheric effect must be corrected for. Existing routine procedures to tackle ionospheric effect are to use dual-frequency observations. Most of the error can be eliminated due to the frequency-dependent nature of the ionospheric effects. This is why both GPS and GLONASS systems use two frequencies to transmit signals. However, the dual-frequency method can only account for the linear component of the ionospheric effect (i.e. the 1st order refractive error), which is sufficient for most navigation applications. There are some applications that require accounting for higher order ionospheric refractive error. The imminent GPS modernization (McDonald, 2002) and GALILEO (European Commission, 2003) deployment provides the potential to further refine the ionospheric models due to the introduction of a third-frequency.

During the periods of high Total Electron Content (TEC), higher-order ionospheric effects can reach several tens of centimetres in range error, which adversely affect the accuracy of GPS observables. Brunner and Gu (1991) use the full form of the refractive index given in Eq. (3) to calculate the residual range error from the first–order form of refractive index. Their model also includes the effects of the Earth's geomagnetic field and ray bending at both GPS frequencies, L_1 and L_2. However, in order to achieve millimetre-level of accuracy for ionospheric error correction, both the actual maximum electron density, N_m, and the average value of the longitudinal component of the Earth's magnetic field along the ray path are required. For practical usages, however, it is difficult to either access or estimate these parameters.

Bassiri and Hajj (1993) carry out a similar research on high-order GPS ionospheric range errors. The electron density profile shape is required in the model and higher-order ionospheric corrections are not as good as those claimed by Brunner and Gu (1991). However, their models are much easier to be implemented since the actual ionospheric data is required.

In this paper, a triple-frequency method of correcting both the 1st and the 2nd order ionospheric refraction is presented. In addition, a triple-frequency ionosphere-free combination method is also developed for the imminent GPS modernization and GALILEO systems. The 3rd order ionospheric refraction correction is also investigated in a simplified way. It is expected that a millimetre level of ionospheric correction accuracy can be achieved using the methods developed.

2 Ionosphere delay error

The ionosphere is a dispersive media that is ionized by the ultraviolet radiation from the sun. The TEC value of the ionosphere varies mainly due to day-to-night variations. However, it also depends on a user's location (i.e. the geomagnetic latitude), time of year, and sunspot cycle. The ionosphere affects the GPS signals propagating through it. The propagation of GPS signals has the characteristics that there is a phase advance of the carrier and a group delay of the code. The phase velocity v_p and the velocity of the group of frequencies v_g have the following relationship:

$$v_g = v_p - \lambda \frac{dv_p}{d\lambda}$$

Incorporating $d\lambda / \lambda = -df / f$ in to the above equation, we then have

$$v_g = v_p + f \frac{dv_p}{df} \tag{1}$$

The refractive index of GPS signals is defined as:

$$n = \frac{c}{v} = \frac{\lambda_{vac}}{\lambda}$$

Where c and λ_{vac} are the velocity and wavelength of GPS signals in vacuum respectively. v and λ are the velocity and wavelength of GPS signals in the ionosphere respectively. Substitute the above equation into equation (1), then we obtain

$$n_g = n_p / \left[1 - (\frac{dn_p}{df}) / (\frac{n_p}{f}) \right]$$

Develop the above equation into a series form of $(1 - x)^{-1} = 1 + x - x^2 + \cdots$, and take the first two terms only. The relation of n_g and n_p can then be expressed as

$$n_g = n_p + f \frac{dn_p}{df} \tag{2}$$

According to Brunner and Gu (1991), the diffused refractive index can be expressed as

$$n_p = 1 - C_X N_e f^{-2} / 2 \pm C_X C_Y N_e (H_0 \cos\theta) f^{-3} / 2 - C_X{}^2 N_e^2 f^{-4} / 8 \tag{3}$$

Where

$$\begin{cases} C_X = \dfrac{e^2}{4\pi^2 \varepsilon_0 m} \\ C_Y = \dfrac{\mu_0 e}{2\pi m} \end{cases}$$

and e represents the power of an electron, m is the mass of an electron, N_e is the electron density in the atmosphere, H_0 is the quantity of the geomagnetic field, Θ is the angle between H_0 and the direction of signal transmitting, f is the frequency, ε_0 and μ_0 are physical constants. Equation (3) defines the refractive index up to an accuracy of 10^{-9} (Brunner and Gu, 1991), so it ensures that the optical path length is of millimetre accuracy.

Equation (3) can then be expressed as

$$n_p = 1 + a_1 / f^2 + a_2 / f^3 + a_3 / f^4 \tag{4}$$

Where a_1, a_2 and a_3 are the simplified coefficients. Substituting equation (4) into equation (2), we have

$$n_g = 1 - a_1 / f^2 - 2a_2 / f^3 - 3a_3 / f^4 \tag{5}$$

When GPS signals penetrate through the ionosphere, the distance and phase errors of the transmission paths due to the variation of the refractive index:

$$\begin{cases} \delta\rho = \int_s (n-1)ds \\ \delta\varphi_p = \frac{f}{c}\int_s (n_p - 1)ds \end{cases} \tag{6}$$

Substituting equations (4) and (5) into equation (6) gives

$$\begin{aligned} \delta\rho_p &= \int_s (n_p - 1)ds \\ &= \int_s (a_1/f^2 + a_2/f^3 + a_3/f^4)ds \\ &= \int_s (a_1/f^2)ds + \int_s (a_2/f^3)ds + \int_s (a_3/f^4)ds \end{aligned} \tag{7}$$

$$\begin{aligned} \delta\rho_g &= \int_s (n_g - 1)ds \\ &= \int_s (-a_1/f^2 - 2a_2/f^3 - 3a_3/f^4)ds \\ &= -\int_s (a_1/f^2)ds - 2\int_s (a_2/f^3)ds - 3\int_s (a_3/f^4)ds \end{aligned} \tag{8}$$

Again, ionospheric refractive error is generally about 15 meters in the daytime, and about 3 m in the evening, and maximum 50 m in the zenith, 150 m in the horizon (Zhou, 1995). If only the 1st-order ionospheric refractions are corrected, the remaining error is usually about 1.5 centimetres, and its maximum is about 5~15 centimetres. If better than 1 centimetre accuracy is required, we must correct the 2nd order ionospheric refraction in theory. Moreover, if you want to achieve millimetre-level of precision, the 3rd order ionospheric refraction must be corrected for. This also suggests that the remaining error after applying the 1st-order ionospheric refraction correction is about centimetre-level for phase measurements and about meter-level for code pseudoranges. This may be much higher when there are severe sunspot activities. It is concluded that for precise positioning applications the 2nd order and the 3rd order ionospheric refractions must be corrected for.

3 Triple-frequency method for 2nd order ionospheric refraction correction

It is well known that the 1st order ionospheric refractions can be eliminated by current dual-frequency method using its frequency dependent nature. With GPS modernization, it becomes possible to correct 2nd order ionospheric refraction using three frequency measurements. Below we present the procedure of resolving the 2nd order ionosphere effects.

From equation (7), we have (only the 1st- and 2nd-order ionospheric refractions are taken into account here)

$$\delta\rho_p = \int_s (a_1/f^2 + a_2/f^3)ds = A_1/f^2 + A_2/f^3$$

Where,

$$A_1 = \int_s a_1 ds\,,\quad A_2 = \int_s a_2 ds$$

To distinguish the effects from different frequencies, the above equation can be re-written as

$$\delta\rho_p(f_i) = A_1/f_i^2 + A_2/f_i^3 \qquad i = 1,2,3 \tag{9}$$

The 1st order ionospheric refraction removed combinations can be obtained as

$$\delta\rho_p(f_1)\bullet f_1^2 - \delta\rho_p(f_2)\bullet f_2^2 = A_2(1/f_1 - 1/f_2)$$
$$\delta\rho_p(f_1)\bullet f_1^2 - \delta\rho_p(f_3)\bullet f_3^2 = A_2(1/f_1 - 1/f_3)$$

Then the 2nd-order ionospheric refraction removed combinations is

$$\frac{(\delta\rho_p(f_1)\bullet f_1^2 - \delta\rho_p(f_2)\bullet f_2^2)(f_1 f_2)}{f_2 - f_1} - \frac{(\delta\rho_p(f_1)\bullet f_1^2 - \delta\rho_p(f_3)\bullet f_3^2)(f_1 f_3)}{f_3 - f_1} = 0$$

This can be simplified as

$$\begin{aligned} & f_1^3(f_3 - f_2)\bullet\delta\rho_p(f_1) + f_2^3(f_1 - f_3)\bullet\delta\rho_p(f_2) \\ &\qquad + f_3^3(f_2 - f_1)\bullet\delta\rho_p(f_3) \\ &= B_1\bullet\delta\rho_p(f_1) + B_2\bullet\delta\rho_p(f_2) + B_3\bullet\delta\rho_p(f_3) \\ &= 0 \end{aligned}$$

Where,

$$\begin{cases} B_1 = f_1^3(f_3 - f_2) \\ B_2 = f_2^3(f_1 - f_3) \\ B_3 = f_3^3(f_2 - f_1) \end{cases}$$

Given $m_i = f_I \cdot B_i / (B_1 + B_2 + B_3)$, i=1,2,3, the above equation becomes:

$$m_1\bullet\delta\rho_p(f_1) + m_2\bullet\delta\rho_p(f_2) + m_3\bullet\delta\rho_p(f_3) = 0$$

This means that the 2nd order ionospheric refractions have been corrected and this procedure is also suitable for code measurements. Considering the following relationship

$$\delta\varphi_p = \frac{f}{c}\delta\rho_p\,,$$

we obtain:

$$\delta\varphi_p(f_1)\bullet m_1/f_1+\delta\varphi_p(f_2)\bullet m_2/f_2+\delta\varphi_p(f_3)\bullet m_3/f_3$$
$$=\delta\varphi_p(f_1)\bullet m+\delta\varphi_p(f_2)\bullet n+\delta\varphi_p(f_3)\bullet k$$
$$=0$$

where $m = m_1/f_1$, $n = m_2/f_2$, $k = m_3/f_3$.

Then the 1st-order and 2nd order ionospheric refraction removed combination of three-frequency can be expressed as:

$$\varphi = m\varphi_1 + n\varphi_2 + k\varphi_3 \tag{10}$$

The corresponding phase combination is

$f = m f_1 + nf_2 + kf_3$

$\lambda = c/f$

The relations between the ionospheric correction and phase pseudorange observation are given below. The phase pseudorange observation equations are

$$\rho_i = \rho_0 + \delta\rho_p(f_i) = \rho_0 + A_1/f_i^2 + A_2/f_i^3 \quad (i = 1, 2, 3) \tag{11}$$

From the above equations we obtain

$$A_1 = \frac{\rho_{12} f_1^3 (f_3^3 - f_2^3) - \rho_{23} f_3^3 (f_2^3 - f_1^3)}{f_1^3 (f_2 - f_3) + f_2^3 (f_3 - f_1) + f_3^3 (f_1 - f_2)} \tag{12}$$

$$A_2 = -\frac{\rho_{12} f_1^3 f_2 f_3 (f_3^2 - f_2^2) - \rho_{23} f_1 f_2 f_3^3 (f_2^2 - f_1^2)}{f_1^3 (f_2 - f_3) + f_2^3 (f_3 - f_1) + f_3^3 (f_1 - f_2)} \tag{13}$$

In GPS modernization, the three GPS signal frequencies are L$_1$ ： 1575.42MHz （154×10.23MHz）； L$_2$ ： 1227.60MHz（120×10.23MHz）and L$_5$：1176.45MHz（115×10.23MHz） respectively. Substituting these values into equations (12), (13) and (9) gives

$$\begin{cases} \delta\rho_p(f_1) = -6.080583\rho_{12} + 20.049766\rho_{23} \\ \delta\rho_p(f_2) = -7.080583\rho_{12} + 20.049766\rho_{23} \\ \delta\rho_p(f_3) = -7.080583\rho_{12} + 19.049766\rho_{23} \end{cases} \tag{14}$$

where, $\rho_{12} = \rho_1 - \rho_2$, $\rho_{23} = \rho_2 - \rho_3$. These are the formulas of calculating the value of ionospheric refraction by three-frequency phase measurements. GPS code pseudorange measurements can be obtained in a similar way.

The 1st order ionospheric refraction takes up 99% of the total effects. Table 1 lists the remaining ionospheric refraction with both the 1st order and 2nd order terms corrected. It is demonstrated that correction of the 2nd order term is necessary for a precision of better than one centimetre-level. Correcting 3rd-order term is also necessary for the precision of more than one millimetre level. Here we assume that the sum of 4 terms at the right hand side of equation (3) is the total of the ionospheric refraction.

Table 1. Maximum vertical ionospheric range error [unit=m]

CASE 1 : TEC=4.55e18m^{-2} N_m =20.0e12m^{-3}			
Frequency	1st-order effect (1/f^2)	2nd-order effect (1/f^3)	3rd-order effect (1/f^4)
L1	73.8428	0.0818	0.0079
L2	121.6150	0.1729	0.0215
L5	132.4201	0.1964	0.0254
L1/L2	0	0.0590	0.0130
L1/L2/L5	0	0	0.0054
CASE 2 : TEC=1.38e18 m^{-2} N_m =6.0e12 m^{-3}			
Frequency	1st-order effect (1/f^2)	2nd-order effect (1/f^3)	3rd-order effect (1/f^4)
L1	22.3963	0.0248	0.0007
L2	36.8854	0.0524	0.0020
L5	40.1626	0.0596	0.0023
L1/L2	0	0.0179	0.0012
L1/L2/L5	0	0	0.0005

Note that CASE 1 corresponds to an absolute maximum solar cycle condition that rarely happens, while CASE 2 is typical for high N_m values that are frequently observed (Klobuchar, 1983).

4 Third order ionospheric refraction correction

From Table 1, it can be seen that the 3rd order effect is still as large as 25mm in carrier L$_5$. The ionospheric free linear combination of the phase observation on L$_1$, L$_2$, and L$_3$ still remains 5mm in CASE 1. Thus, we must correct the 3rd order effects.

In order to calculate the 3rd order term, an integration of N_e^2 along the signal path is required. The integration is very difficult and involves a complicated process. Hartmann and Leitinger (1984) have suggested a shape parameter η through the following relation:

$$\eta \equiv \int_s N_e^2 ds \Big/ N_m \int_s N_e ds$$

Where N_m is the maximum value of the electron density N_e. The value of the shape parameter only slightly varies with the elevation angle and the maximum electron density. This is proven by Hartmann and Leitinger (1984) and Brunner (1991). According to their research, η can be assigned an approximate value of 0.66 and this represents η for any profile of the electron density distribution in the ionospheric with sufficient accuracy. The 3rd order effect of ionosphere can be expressed as:

$$\delta\rho_p^{3rd-order} = -C_X{}^2 \eta N_m/(8f^4)\cdot\int_s N_e ds \tag{15}$$

If we select the semi-empirical ionospheric model developed by Anderson et al. (1987) who gives an electron density distribution in the ionosphere. The approximate relation of N_m and TEC can be expressed as

$$\mathrm{TEC}=k\cdot N_m \tag{16}$$

Where, $k \approx 2.27\times 10^5$m.

TEC can be obtained in several ways; one convenient way is to obtain it from the difference between ρ_1 and ρ_2. Neglecting high-order effects, we have

$$\delta\rho_p(f_1) = -1.54573\rho_{12} = -C_X \cdot TEC/(2f_1{}^2)$$

We then have

$$\mathrm{TEC}=9.51768\times 10^{16}\rho_{12} \tag{17}$$

$$\delta\rho_p{}^{3-order} = -C_X{}^2\eta/(8kf^4)\cdot(TEC)^2$$

$$= -2.13964\times 10^{31}\rho_{12}{}^2/f^4 \tag{18}$$

Using the difference between ρ_1 and ρ_2, the 3rd order ionospheric effect can be easily calculated using equation (18). Although some approximate parameters are adopted in equation (18), it is still of high accuracy. Its relative error is less than ±5%. Considering the fact that the 3rd order effects are generally less than 10mm, the absolute accuracy of equation (18) is better than ±1mm.

5 Conclusions

The availability of the third frequency from the modernized GPS and GALILEO systems provides an opportunity to eliminate the ionospheric propagation effects more efficiently. In this paper, a three frequency ionosphere free combination is given, which has effectively eliminated the 1st order and the 2nd order ionospheric effects. Furthermore, formulas of calculating the value of ionospheric refraction by differencing the carrier phase measurements in three frequencies are derived. Finally, an equation calculating the 3rd order ionospheric effects using the difference between ρ_1 and ρ_2 is given. Although some approximate parameters are adopted in the equation, it is still of high accuracy. The absolute accuracy of the 3rd order ionospheric effects is better than ±1mm.

The advantage of the triple frequency methods for correcting high-order ionospheric refraction is apparent to the imminent GPS modernization program. It is concluded that the proposed triple frequency methods can correct the ionospheric refraction at millimetre-level. Since the corrections are given in simple forms, these methods can be easily implemented in many real-time applications to considerably eliminate the error in current ionospheric models. We believe that this will potentially contribute to a better long-range baseline ambiguity resolution and an accuracy improvement in precise point positioning. In addition, the sensitivity of detecting cycle slips by using a refined ionospheric refraction can be improved. These new methods are also applicable to GALILEO system as well.

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